

# A Consideration on the Directions of Capital Movements under Capital Market Imperfections

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## Abstract

I investigate capital movements under capital market imperfections in an endogenously growing economy. According to the classical or standard model of international monetary theory, capital flows from rich to poor countries because marginal productivity of capital that equals to interest rate in poor countries is higher than that in rich countries. In the actual world, however, the capital in most of developing countries flows out to the foreign financial market. In this paper, I show that the interest rates depend on the monitoring cost or verification cost which is the cost to verify or to monitor the realized return of borrowers' investment projects. Under then assumption that verification cost depends on the existing capital in an economy, the relation between verification cost and interest rate determines the direction of capital movements. If growth of economy leads to the decreases in the cost of verification, capital in developing countries flows out to developed countries.

Keywords : Capital Market Imperfections ; Capital Movements ; Verification Cost

JEL Classification Number : E44 ; F21

## 1. Introduction

Capital Flight is a much important problem in developing countries. Capital flight leads to a decrease in the funds that are sources of domestic investments.

According to the classical or standard model of international monetary theory, capital flows from rich to poor countries because marginal productivity of capital that equals to interest rate in poor countries is higher than that in rich countries. However, in the actual world, the capital in most of developing countries flows out to the foreign financial market, especially, to European financial market<sup>1)</sup>.

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<sup>1</sup> The detail structure of capital flight is shown in Kuroyanagi and Hamada(1993)

The approach of capital market imperfections has widely recognized under the world where there is an asymmetric information between lenders and borrowers. By incorporating the concept of capital market imperfections into macroeconomics, that approach are able to explain some economic phenomenon which cannot be understood by standard theory (for example, see Banerjee and Newman(1988), Galor and Zeira(1988), Williamson(1989) and so on)<sup>2</sup>. It is shown in this paper that how the capital movements occur under capital market imperfections in an endogenous growth model, which is a variant of Romer(1986) and Lucas(1988): externalities give rise to a production function with increasing return to scale. Gertler and Rogoff(1990) provides a model of international capital movement with asymmetric information between lenders and borrowers. Gertler and Rogoff( 1990) has shown that net worth of borrowers in rich countries may work to reduce or possibly offset capital outflows to poor countries. Also Hamada and Sakuragawa(1993) which develops Gertler and Rogoff to a dynamic model shows that too hasty liberalization of domestic capital market in developing country may worsen the long-run state of country through flight of capital.

I provide a model which introduces the endogenously growing economy into the costly-state-verification approach developed by Townsend(1979). Under the assumption that verification cost depends on the existing capital in an economy, the relation between verification cost and interest rate determines the direction of capital movements. The important implication of this paper is that if an increase in capital in an economy leads to a decrease in verification cost, capital flows from developing country with less capital to developed country.

Next section provides a basic endogenous growth model with contract problem,section 3 discuss two countries case and analyzes the capital movements.

## 2. The Model

Consider two-period overlapping-generations model. The economy is composed countable infinity agents. There is no population growth. Agents are either lenders or entrepreneurs, where a fraction  $\alpha$  of agents is a lender, and  $1 - \alpha$  fraction of agents is an entrepreneur. There are two types of goods: the one is consumption good, the other is capital good. The price of consumption good is numeraire. Both agents are protected by liability constraint.

Each lender is endowed with one unit of labor force, and he supplies inelastically his labor force to a firm owner and can earn the wage at the first period of his life. Each lender born at period  $t$  maximizes the expected value of the utility  $u(c_t, c_{t+1}, e_{t+1}) = c_t + \frac{1}{\rho}(c_{t+1} - e_{t+1})$  where  $c_i$  is consumption at  $i$  ( $i = t, t+1$ ),  $e_{t+1}$  is the quantity of effort expended to verify entrepreneur's (or firm owner's) returns, and  $\rho$  is the subjective gross discount rate. All lenders have an unbounded effort at period  $t+1$ .

Each lender differs in the subjective discount rate, which follows a continuously differentiable

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<sup>2</sup> Sakuragawa and Hamada(1992) and Pagano(1993) provide the survey of relation between capital market imperfections and economic development.

probability distribution function  $H(\rho)$  with a positive density  $h(\rho)$  over the support  $[0, \infty]$ . Let  $r_{t+1}$  denote the interest rate prevailing between period  $t$  and  $t+1$ , which will be determined endogenously. In period  $t$  each lender chooses whether to save or to consume the wage income at period  $t$ . Any lenders who face  $p$  equal to or less than  $r_{t+1}$  lends the wage income to others, while anyone who faces  $p$  more than  $r_{t+1}$  does not lend to others and consume all the wage income at the first period of his life. As a result,  $H(r_{t+1})$  is the measure of lenders who lend to others, while  $1-H(r_{t+1})$  is the measure of lenders who never lend to others.

Each entrepreneur consumes only in period  $t+1$  and maximizes the expected consumption. Each entrepreneur receives no endowment in either period, but has access to only one investment project at period  $t$  and has production technology of consumption good at the beginning of period  $t+1$ .

The investment project at period  $t$  produces the capital good (namely physical capital) that will be able to input into entrepreneurs' consumption good production at period  $t+1$ . The production technology of investment good is AK-type under which one unit of input at the beginning of period  $t$  yields an uncertain  $\eta$  units of output at the end of period  $t$ .  $\eta$  is the project-specific shock and  $\eta$  follows a uniform distribution over the support of  $[0, \theta]$  with a mean of  $\theta$ . For simplicity, we assume that  $A=1$ .

Although the distribution of  $\eta$  is public information, actual realization of  $\eta$  is private information. Each entrepreneur is able to observe freely but lenders need a verification costs in order to observe entrepreneur's actual output.

Lenders must expend  $B(I_t)$  unit of effort as the verification costs in order to observe entrepreneur's actual realization of project when an amount of inputs is  $I_t$ .  $B' > 0$  or  $B' < 0$ <sup>3</sup>.

In period  $t+1$  entrepreneurs will become firm-owners who produce the consumption good using the Cobb-Douglas type production technology with Marshallian externality:

$Y_{t+1} = F(K_{t+1}, L_{t+1}) \equiv \bar{k}_{t+1}^{1-\gamma} K_{t+1}^\gamma L_{t+1}^{1-\gamma}$ , where  $\bar{k}_{t+1}$  is the average capital stock per firm at period  $t+1$ . Rearranging to per capita term, we obtain

$$y_{t+1} = \bar{k}_{t+1}^{1-\gamma} k_{t+1}^\gamma, \text{ where } y_{t+1} \equiv Y_{t+1} / L_{t+1} \text{ and } k_{t+1} \equiv K_{t+1} / L_{t+1}$$

Each firm-owner behaves competitively taking  $\bar{k}_{t+1}$  as given. Considering an equilibrium in which  $\bar{k}_{t+1} = k_{t+1}$ , is satisfied, competitive factor markets require that

$$(1) \quad w_t = (1 - \gamma)k_t, \quad (2) \quad r_{t+1} = \gamma$$

where (1) implies that the marginal product of labor is equal to the wage rate, and (2) implies that the marginal product of capital is equal to the interest rate. Capital depreciates fully after one period.

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<sup>3</sup> This assumption of the relation between verification cost and input size (equal to existing capital in next period) is important factor which determines the direction of capital movements in section 3.

## 2-1. Contracting Problem

In period  $t+1$  entrepreneurs become firm owners with capital good. Capital good at period  $t+1$  is the output of project operated at period  $t$ . However, since entrepreneurs do not have endowment they have to borrow the funds to operate their project. Since the realization of project is private information, lenders have to write a contract with entrepreneurs.

We assume that the firm owner repays his promised interest for lenders after production of consumption good at period  $t+1$ .

Since the production technology is  $y_{t+1} = \bar{k}_{t+1}^{1-\gamma} k_{t+1}^\gamma$ , and the amount of capital good is  $\eta I_t$  when the size of inputs of investment project at period  $t$  is  $I_t$ , the firm-owner will be able to receive  $\gamma k_{t+1} = \gamma \eta I_t$  as the production return at period  $t+1$ .

The optimal contract between lenders and borrowers is the debt contract shown by Gale and Hellwig(1981) and Williamson(1986). Suppose that  $S$  is a set in which verification occurs and  $Sc$  is the complement.  $\eta \in Sc$ , then verification occurs, and if  $\eta \in S$ , then it does not occur. Since the entrepreneur receives  $\gamma \eta$  units of consumption good per unit of investment after production of consumption good at the beginning of period  $t+1$ , then we can consider a gross loan interest rate  $R_{t+1}$  satisfying that

$$(3) \eta \in S, \quad \text{if } \gamma \eta < R_{t+1} \text{ and } (4) \eta \in Sc, \quad \text{if } \gamma \eta \geq R_{t+1}$$

From (3) and (4) we are able to consider that a cut off  $\gamma \eta^* = R_{t+1}$  exists.

Whenever  $\gamma \eta \geq R_{t+1}$  the entrepreneur reports  $\eta = \eta^*$ , and pays  $R_{t+1}$  per unit, while whenever  $\gamma \eta < R_{t+1}$ , the entrepreneur announces  $\eta$  truthfully, verification occurs, and the lenders receive all the profits.

The optimal problem between an entrepreneur who borrows  $I_t$  units of consumption good and a lender is as follows:

$$(5) \max_{R_{t+1}, q_t} \int_{R_{t+1}/\gamma}^{\bar{\eta}} [\gamma \eta I_t - R_{t+1} I_t] \frac{1}{2\theta} d\eta$$

subject to

$$(6) \int_0^{R_{t+1}/\gamma} \gamma \eta I_t \frac{1}{2\theta} d\eta + R_{t+1} I_t (1 - \frac{R_{t+1}}{2\theta\gamma}) - \beta(I_t) \frac{R_{t+1}}{2\theta\gamma} = I_t r_{t+1}$$

Using (6), (5) is rewritten to

$$(7) I_t \{ \gamma \mu - r_{t+1} - \beta(I_t) \frac{R_{t+1}}{2\theta\gamma} \}$$

As a result, (5) and (6) are replaced by

$$(8) \quad \max_{\eta^*} \int_0^{2\theta} (\gamma\eta I_t - I_t R_{t+1}) \frac{1}{2\theta} d\eta$$

subject to

$$(9) \quad \pi(R_{t+1}) \equiv \int_0^{\eta^*} \gamma\eta I_t \frac{1}{2\theta} d\eta + R_{t+1} I_t (1 - \frac{1}{2\theta\gamma}) - \beta(I_t) \frac{R_{t+1}}{2\theta\gamma} = I_t r_{t+1}$$

Considering only the equilibrium in which all entrepreneurs receive equal loans, and from (5) and(6),

$$\eta^* = \frac{R_{t+1}}{\gamma}.$$

$$(10) \quad \int_0^{\frac{2\theta}{\gamma}} \gamma\eta I_t \frac{1}{2\theta} d\eta + R_{t+1} I_t (1 - \frac{1}{2\theta\gamma}) - \beta(I_t) \frac{R_{t+1}}{2\theta} = R_{t+1} I_t - \frac{1}{4\theta\gamma} R_{t+1}^2 I_t - \frac{\beta(I_t) R_{t+1}}{2\theta\gamma} = I_t r_{t+1}$$

Differentiating (10) with respect to  $R_{t+1}$ , we obtain

$$(11) \quad \pi'(R_{t+1}) = I_t - \frac{I_t R_{t+1}}{2\theta\gamma} - \frac{\beta(I_t)}{2\theta\gamma}$$

Study now the determine the equilibrium. We impose the following restriction on parameter value to guarantee an equilibrium with positively-valued variables.

Assumption  $\beta' < 2\theta\gamma$

Therefore,  $\pi'(R_{t+1}) < 0$  for some  $R_{t+1} \in [0, 2\theta]$ . Under the Assumption,  $\pi(R_{t+1})$  is strictly concave in  $R_{t+1}$ . It is useful to specify the pair of interest rate which maximizes the lender's profits, so from (11), we obtain

$$(11') \quad \pi'(R_{t+1}) = I_t - \frac{I_t R_{t+1}}{2\theta\gamma} - \frac{\beta(I_t)}{2\theta\gamma}$$

In this economy, since there is an infinite competitive entrepreneur, no one can earn excess profit. The zero-profit condition for the entrepreneur is

$$(12) \quad r_{t+1} = \gamma\theta - \beta(I_t) \frac{R_{t+1}}{2\theta\gamma}$$

The market clearing condition for capital is

$$(13) \quad (1 - \alpha)I_t = \alpha H(r_{t+1})(1 - \gamma)k_t$$

where L.H.S of (13) implies aggregate demand for loans and R.H.S of (13) represents aggregate supply of loans.

### Definition

There are two types of equilibria: one is no-rationing equilibrium (NRE) and the other is associated with credit rationing (RE).

1. An NRE is a sequence of endogenous variables  $\{I_t, r_t, R_t, w_t, k_t\}_{t=0}^{\infty}$ , satisfying (1), (2), (10), (12) and (13), where  $k_0$  is given.

2. An RE is a sequence of endogenous variables  $\{I_t, r_t, R_t, w_t, k_t\}_{t=0}^{\infty}$ , satisfying (1), (2), (10), (11') and (13), where  $k_0$  is given.

Hereafter, only consider the economy in RE. Since our purpose is to show the relation economic growth and capital market imperfections. From (11'),  $R_{t+1} = 2\theta\gamma - \frac{\beta(I_t)}{I_t}$ . And incorporate this result

into (10),  $r_{t+1} = \theta\gamma - \beta(I_t) + \frac{\beta(I_t)^2}{4\theta\gamma}$ . The equilibrium is characterized by the pair

$$\{R_{t+1}, r_{t+1}\} = \left\{ 2\theta\gamma - \frac{\beta(I_t)}{I_t}, \theta\gamma - \beta(I_t) + \frac{\beta(I_t)^2}{4\theta\gamma} \right\}$$

which maximizes the intermediary's profit, given the interest rate  $r_{t+1}$ . The figure 1 illustrates the intermediary's profit function and entrepreneur's zero-profit condition (downward-sloping solid straight line). The equilibrium is not determined by the intersection of  $r_{t+1} = \pi(R_{t+1})$  and (12), denoted by F. The equilibrium is determined by point E. Since, at this point, intermediary's profit is maximized, intermediary has no incentive to raise the loan interest.

Note that (12) is irrelevant to the determination of equilibrium, but rather the entrepreneur earns

$$\theta\gamma - r_{t+1} - \frac{\beta(I_t)}{4\theta\gamma} R_{t+1} = \frac{\beta(I_t)^2}{4\theta\gamma}$$

per unit of capital. The entrepreneur is credit rationed in the sense that he would be willing to pay higher loan interest rates in order to apply for a greater amount of loan, but the offer is rejected. The credit ration

is summarized as follows.

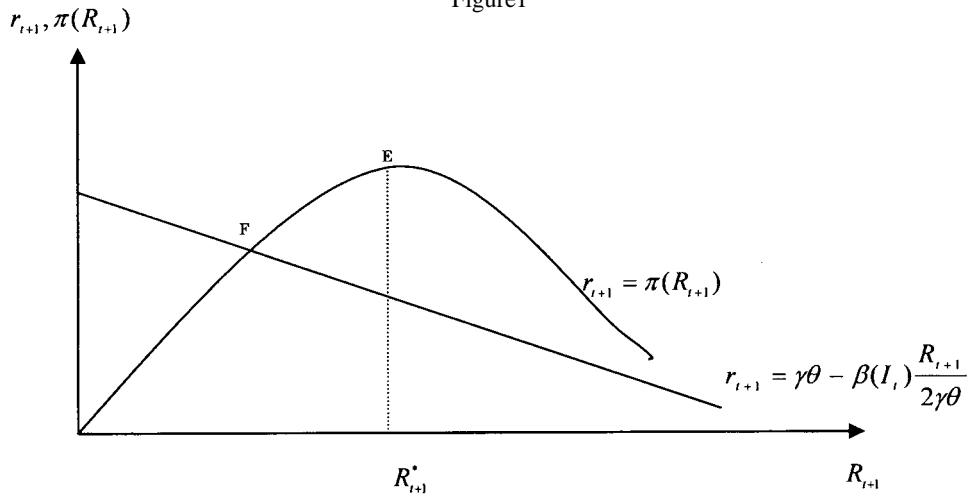
**Proposition 1**

Consider the pair  $\{R^*, r^*\}$  which satisfies (10), (11) and (13).

If the pair  $\{R^*, r^*\}$  satisfies  $\gamma\mu < r_{t+1}^* + \beta G(R_{t+1}^* / \gamma)$ , then equilibrium is no-rationing equilibrium (NRE).

If the pair  $\{R^*, r^*\}$  satisfies  $\gamma\mu \geq r_{t+1}^* + \beta G(R_{t+1}^* / \gamma)$ , then equilibrium is associated with credit rationing (RE).

Figure1



**2-2. On Growth Rate**

Consider only the economy in RE. The amount of capital per capita existing on average at the beginning of period t+1,  $k_{t+1}$ , is  $\theta I_t^*$

$$(14) \quad k_{t+1} = \theta I_t^*$$

Incorporate (13) into (14) and then we obtain  $k_{t+1} = \frac{\alpha(1-\gamma)\mu}{1-\alpha} H(r_{t+1}^*) k_t$ . It follows from

$$r_{t+1} = \theta\gamma - \beta(I_t) + \frac{\beta(I_t)^2}{4\theta\gamma} \text{ in RE,}$$

$$k_{t+1} = \frac{\alpha(1-\gamma)\theta}{1-\alpha} H(r_{t+1}^*) k_t = \frac{\alpha(1-\gamma)\theta}{1-\alpha} H\left\{\theta\gamma - \beta(k_{t+1}/\theta) + \frac{\beta(k_{t+1}/\theta)^2}{4\theta\gamma}\right\} k_t$$

It is difficult to show the dynamic process of growth rate.

### 3. Capital Movements

The purpose of this section is to show the capital movements under which capital markets are opened between two countries. Only the difference in two countries is the amount of existing capital: one is developed country with much capital, the other is developing country with less capital. Capital movements are shown to compare the interest rate in both countries.

At first, we examine the case of which capital accumulation leads to an increase in the verification costs:  $\beta' < 0$ . In this case, the verification cost in developed country is more expensive than that in developing country. Differentiation of the interest rate with respect to the existing capital,

$$\frac{\partial r_{t+1}}{\partial k_{t+1}} = \frac{\partial r_{t+1}}{\partial \beta} \frac{\partial \beta}{\partial k_{t+1}} = \left(-\beta' + \frac{\beta'}{2\theta\gamma}\right)\beta' < 0,$$

under Assumption. This implies that the interest rate in developed country higher than that in developing country and capital flows from developed country to developing country.

Next case is that capital accumulation leads to a decrease in verification cost:  $\beta' < 0$ . We can see the result by same method of first case.

$$\frac{\partial r_{t+1}}{\partial k_{t+1}} = \frac{\partial r_{t+1}}{\partial \beta} \frac{\partial \beta}{\partial k_{t+1}} = \left(-\beta' + \frac{\beta'}{2\theta\gamma}\right)\beta' < 0,$$

under Assumption  $\beta < 2\theta\gamma$ . This implies that the interest rate in developed country lower than that in developing country and capital flows from developing country to developed country.

#### Proposition 2

1. Suppose that capital accumulation leads to an increase in verification cost. Capital flows from a country with much capital to a country with less capital.
2. Suppose that capital accumulation leads to a decrease in verification cost. Capital flows from a country with much capital to a country with less capital.

### 4. Conclusion

We show that the relation between an amount of existing capital and verification cost would determine the direction of capital movements. An important result is that if the amounts of capital lead to an decrease the verification cost, capital flows from developed country with much capital to developing country with less capital. However, this result almost owes the assumption associated with the relation between an amount of existing capital and verification cost. This must be supported by empirical evidence. This will be my future research.



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