Dynamic Analysis of Non-self-fulfilling Bubbles in an Overlapping Generations Economy

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Abstract

The purpose of this paper is to formulate the growth process of asset bubbles which is consistent with actual experiences of bubbles characterized by the emergence, expansion, and sudden crash and to analyze its effects on dynamic aspects of the economy. We show that if we remove the assumption of perfect foresight of all agents from the standard model of rational self-fulfilling bubbles, such a growth process of bubbles, which we call “non-self-fulfilling bubbles”, can be formulated. We also investigate the endogenous timing of the crash of non-self-fulfilling bubbles, growth and welfare effects of them, and a public policy which cuts the duration of the process of non-self-fulfilling bubbles short.

JEL Classification: E52, O41, O42
Key Words: non-self-fulfilling bubbles, overlapping generations, endogenous growth

1. Introduction

The effects of rational bubbles on dynamic resource allocation of the economy have been explored actively in the context of macroeconomics for last several decades. In 1980’s Wallece(1980), Tirole(1985), O’Connell and Zeldes(1988) etc investigated the theoretical condition under which deterministic bubbles can be positively valued in the framework of the standard neoclassical growth model, and the following two main conclusions were derived: (1) deterministic bubbles can exist in equilibrium growth paths if and only if the economy without bubbles is dynamically inefficient, and (2) an introduction of deterministic bubbles in a dynamically inefficient economy can be Pareto-improving. In 1990’s this problem was reexamined in the context of the endogenous growth theory. The representative researches such as Grossman and Yanagawa(1993), King(1992), King and Ferguson (1993), and Futagami and Shibata(1999,2000) etc showed that under the framework of the endogenous growth theory (1) deterministic bubbles can exist even if the economy is dynamically efficient, and (2) in general an introduction of them into the endogenously growing economy harms the welfare of succeeding future generations because it depresses the equilibrium rate of economic growth.

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In all the above researches the subject of the analysis is rational self-fulfilling bubbles, and there have been no attempts to examine a type of bubbles which does not have the self-fulfilling property and accordingly collapses suddenly at some period under the framework of dynamic macroeconomic models. It seems, however, that the theory of rational self-fulfilling bubbles can not explain the actual experiences of bubbles such as the Japanese economy in the latter half of the 1980's. Previous studies concerning rational self-fulfilling bubbles demonstrate that the patterns of growth of such bubbles are either the followings: (1) a normalized bubble shrinks monotonically over time and the economy converges to the bubbleless equilibrium, or (2) a normalized bubble grows with the economy and it remains in fixed proportion to one another in the steady state [2]. Needless to say, however, the actual process of a bubble economy can be characterized by the emergence, expansion, and sudden crash. This fact implies that a theory of rational self-fulfilling bubbles fails to explain the actuality. Furthermore, using the data on Japanese stock prices Fukuta(1996,2002) empirically shows the hypothesis that rational bubbles exist is rejected [3]. From these I think that an investigation of asset bubbles characterized by no self-fulfilling property is necessary to understand the actual bubble process and its macroeconomic consequences. So in this paper we intend to explore such bubbles in an overlapping generations economy and examine the effects of them on dynamic aspects of the economy.

What we have to do first is to clarify the assumption which prevents the formation of dynamic paths characterized by no self-fulfilling property of bubbles in the previous researches. In this respect we show that it is the hypothesis of perfect foresight of all agents that eliminates such a bubble process in equilibrium. This means, in other words, that once we remove its assumption from the model the formation process of bubbles similar to the actual experiences of bubble economies can be generated. In this paper we call such a bubble "non-self-fulfilling bubble". The main purpose of this paper is to examine when such a non-self-fulfilling bubble bursts and what effects it has on the growth and welfare aspects of the economy. In Section 3.1 we explore the endogenous timing of the crash of non-self-fulfilling bubbles theoretically and show that the duration period of a bubble economy is prolonged by such factors as an increase of the saving rate, labor productivity, and population size. In other words this means that the duration period of a bubble economy tends to be prolonged in an economy with potentially high rate of economic growth. In Section 3.2 we investigate how non-self-fulfilling bubbles affect economic growth and intergenerational welfare. It is demonstrated that even though the initial size of non-self-fulfilling bubbles would be very small, it could have negative impacts on capital accumulation and economic growth in the process of expansion. As a result the welfare of all the future generations are harmed, some generation of whom suffers serious damage of economic welfare due to the huge capital loss brought about by the crash of bubbles. We also explore what policy should be done to shorten the duration of the process of non-self-fulfilling bubbles.

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[2] By way of exception there is a theory of stochastic rational bubbles formulated by Blanchard(1979) and introduced into the overlapping generations model by Weil(1987).

[3] Ito and Iwaisako(1996) also conclude that the stock price increase in the second half of 1989 and the land price increase in 1990 are not explained by any asset pricing model based on fundamentals or rational bubbles.

The organization of the rest of this paper is as follows. In the next section we present the basic model and examine the relation between the hypothesis of foresight ability of agents and the formation of non-self-fulfilling bubbles. Section 3 analyses the various economic aspects of non-self-fulfilling bubbles including the endogenous timing of the crash, growth and welfare effects, and an effective public policy which shortens the duration of the process of irrational bubbles. Section 5 concludes the paper.

2. The Model

The model considered in this paper is basically the same as that of Grossman and Yanagawa(1993) which introduced asset bubbles and endogenous growth into the Diamond(1965)'s model. The economy consists of overlapping generations of identical individuals who live for two periods and have the homothetic utility function. We assume the population size of each generation is L and constant over time. In the first period of their life, individuals earn a wage income by supplying one unit of labor inelastically, and save a part of their wage income for old age. In our economy there are two methods of savings: a physical capital asset and an intrinsically useless paper asset, namely a bubble asset which has a zero market fundamentals. The rate of net return on physical capital at period t is r_{t+1}. Concerning bubble assets each individual acquires a claim to $p_{t+1}m_t$ units of consumption goods in the next period by purchasing m_t units of bubbles at the goods price of p_t each at period t. Under these assumptions the maximization problem the representative young individual at period t faces are formulated as follows.

$$\text{Max } U(c_t^y, c_{t+1}^o) = V[u(c_t^y, c_{t+1}^o)]$$

$$\text{s.t. } c_t^y + [s_t + p_t m_t] = w_t,\ c_{t+1}^o = (1+r_{t+1})s_t + p_{t+1}m_t$$

where c_t^y, c_{t+1}^o, s_t, and w_t denote the consumption when young, the consumption when old, the individual savings for physical capital, and the labor income. The assumption that the utility function is homothetic means that $V(\cdot)$ is a strictly increasing function and $u(\cdot,\cdot)$ is homogenous of degree 1. From above the intertemporal budget constraint can be derived as

$$(1) \quad c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} + \left[p_t - \frac{p_{t+1}}{1+r_{t+1}} \right] \times m_t = w_t$$

Since in this economy there is no uncertainty, the following condition concerning the gross return rate of each asset must be hold in equilibrium.

$$(2) \quad 1+r_{t+1} = \frac{p_{t+1}}{p_t} \text{ (or equivalently, } p_t - \frac{p_{t+1}}{1+r_{t+1}} = 0 \text{)}$$

Hence, the intertemporal budget constraint of the individual is given by the following simple one.

(3) $$c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} = w_t$$

Due to the homotheticity of the utility function, the solution of the above maximization problem is

(4) $$S_t^* = s(r_{t+1}) \times w_t$$

where S_t^* means the optimal total savings. Since a physical capital is a perfect substitute for a bubble asset in this model, only the level of the total savings can be determined and the holding ratio between two assets can not be determined from the above problem.

On the production side of the economy we suppose there are many identical firms and the number of them is normalized to 1 for simplicity. Firms face perfectly competitive markets and maximize their profits. The production function of the firm j is represented as

(5) $$y_t^j = F[k_t^j, A(K_t)l_t^j]$$

which is a variant of Romer(1986)'s formulation. y_t^j , k_t^j K_t and l_t^j represent output of firm j , the physical capital stock employed by firm j , the aggregate capital and the labor force employed by firm j , respectively. We assume that the production function exhibits constant returns to scale, the functional form of the labor productivity function is $A(K_t) = aK_t$, and the aggregate labor force employed at each period is L .

Perfect competition in factor markets implies that the following set of first order conditions is satisfied:

(6.a) $$\frac{\partial y_t^j}{\partial k_t^j} = F_1[k_t^j, aK_t l_t^j] = r_t$$

(6.b) $$\frac{\partial y_t^j}{\partial l_t^j} = aK_t F_2[k_t^j, aK_t l_t^j] = w_t$$

Because of the symmetry among firms, the followings hold in equilibrium.

(7) $$1\times = k_t^j = K_t ,\ 1\times l_t^j = L$$

Substituting (7) into (6.a) and (6.b), we have

(8) $$r_t = r = F_1[1, aL], w_t = aF_2[1, aL] \times K_t$$

The market equilibrium conditions of a physical capital asset and a bubble asset are given respectively by

(9.a) $$K_{t+1} = s_t \times L \quad (s_t = S_t^* - p_t m_t)$$

(9.b) $$m_t \times L = 1$$

where we assume the aggreagte nominal supply of bubble assets is 1 and fixed. Let us denote

(10) $1+\kappa_t \equiv \dfrac{K_{t+1}}{K_t}$, $B_t \equiv p_t m_t L$, and $b_t \equiv \dfrac{B_t}{K_t}$

where $1+\kappa_t$, B_t and b_t are the gross economic growth rate, the total real stock of bubble assets and the real bubble-capital ratio, respectively. From (4), (8), (9.a), (9.b), and (10) we can obtain the dynamic equations of this economy.

(11.a) $b_{t+1} = \dfrac{x_1 b_t}{x_2 - b_t}$

(11.b) $1+\kappa_t = x_2 - b_t$

where we denote $x_1 \equiv 1+\bar{r}$ and $x_2 \equiv s(\bar{r})aF_2 L$. From (11.a) we can see x_2 means the potential growth rate of the economy, that is, the growth rate without bubbles. An increase in the individual savings rate s , the labor productivity a , and the size of the population of each generation L raises the potential growth rate.

[Figure 1 around here]

Figure 1 depicts the phase diagram of (11.a). Suppose that the $b-b$ curve is steeper at the origin than the 45 degree line and that the initial real bubble-capital ratio b_0 is positive. As is obvious from figure 1, in this case b grows monotonically and eventually exceeds the maximum allowable value x_2 in some finite period, which means that the real bubble stock at that period exceeds the optimal aggregate savings in the economy [4]. Since such a dynamic path is not feasible, then the initial value of b must be chosen at $b_0 = 0$, which is the unique value consistent with the balanced growth equilibrium. Accordingly no bubble of any size will be possible if the $b-b$ curve is steeper at the origin than the 45 degree line, or equivalently the following condition is satisfied.

(12) $\left.\dfrac{\partial b_{t+1}}{\partial b_t}\right|_{b_t=0} = \dfrac{x_1}{x_2} > 0$

Here let us think in more detail the economic reason why the initial value of must be chosen at $b_0 = 0$ when its dynamic path is infeasible. Let us denote T the period at which b exceeds x_2 for the first time. If all individuals in this economy are assumed to form the perfect foresight expectations, the young at $T-1$ could foresee this eventuality at $T-1$ and therefore would not purchase bubble assets from the (then) old generation (namely, the generation $T-2$) at that time. Furthermore, since the generation $T-2$ are also assumed to expect the future perfectly, they could know the fact that the generation $T-1$ would not purchase bubbles from them, and so they would not purchase bubbles when young, too. Iterating this inference leads to the conclusion the generation 1 would not purchase bubbles

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[4] From (9.a) we can easily show this.

from the generation 0 at the initial period, too. Hence there is no demand for bubbles at the initial period 0 and accordingly the equilibrium price of bubbles drops to zero if all the individuals have the perfect foresight ability. This is the intuitive economic reason why infeasible paths that the real bubble-capital ratio b will diverge must be eliminated in the perfect foresight model.

Above discussion indicates that the dynamic paths which start from b_0 larger than zero may not be excluded if we assume instead agents have only imperfect foresight ability. For example, if we assume that all agents can forecast the situations of the future for only one period ahead, the generation $T-2$ could not know the consequence that the real bubble-capital ratio b exceeds the maximum allowable value x_2 at T , and hence she might purchase bubbles from the generation $T-3$ in her young time. If so, bubbles would not collapse in the initial period even under the condition of (12). In this paper we call such paths the processes of “irrational bubbles”, along which bubbles grow rapidly and burst suddenly at some period which lies between 0 and T . In other words we can define an irrational bubble as a bubble which could not exist provided that the assumption of perfect foresight would be adopted. We argue the various properties of this type of bubbles in the next section.

3. Dynamic Analysis of Irrational Bubbles

In this section we investigate the dynamic properties of irrational bubbles, that is, the endogenous timing of the crash, the growth and welfare effects, and the relation between the duration of the process of irrational bubbles (namely, a bubble economy) and the public policy. Since we focus on the analysis of not a rational bubble but an irrational one, we suppose throughout this section that the condition (12) is satisfied.

3.1 The Endogenous Timing of Collapsing Irrational Bubbles

First of all, we study the endogenous timing of bursting irrational bubbles. As is mentioned in the previous section, the period of the crash must lie between 0 and T if perfect foresight is not assumed, so what we have to do first is to derive the endogenous period T . It seems very difficult, however, to derive the strict solution without any devices for simplicity, so we decide to obtain an approximate solution by linearly approximating the $b-b$ curve in figure 1 at the origin. The approximated $b-b$ curve is given by

(13) $$b_{t+1}=\frac{x_1}{x_2}\times b_t$$

[Figure 2 around here]

In the following we consider (13) as a proxy of (11.a). Suppose that at period 0 the old agents (the generation –1) intend to sell the young (the generation 1) ε unit of bubble assets in terms of the real bubble-capital ratio, and the young willingly consents to that. If such an intergenerational trade would be repeated over time, the real bubble-capital ratio b at T would be $(x_1/x_2)^T\times\varepsilon$. Here we denote T^*

as the value which satisfies the following equation

(14) $$(x_1 / x_2)^T \times \varepsilon = x_2 ,$$

and $\overline{T}^*$ as one which raises the decimals of T^* to a unit. Evidently $\overline{T}^*$ is the period at which b exceeds x_2 for the first time, and the solution of the equation (14) is

(15) $$T^* = \frac{\log(x_2 / \varepsilon)}{\log(x_1 / x_2)}$$

Clearly the irrational bubble must crash before $\overline{T}^*$.

Next we examine the exact timing of the crash. For this purpose we first consider the simplest case where each agent can forecast the situations of the future for only one period ahead. Here let us define the foresight ability ω as follows. That the agent's foresight ability is ω means that she can foresee the future for ω periods ahead. Under this definition the foresight ability of the individual who can only foresee the situations in the next period is $\omega = 1$, and the assumption of perfect foresight corresponds to the case of $\omega = \infty$. In the case where the foresight ability of all the individuals in the economy is $\omega = 1$ the generation $\overline{T}^* - 1$ would not purchase bubbles at $\overline{T}^* - 1$ from the generation $\overline{T}^* - 2$, because the generation $\overline{T}^* - 1$ could anticipate that at the next period b exceeds x_2 and then they could not sell out their bubbles to the generation $\overline{T}^*$. To the contrary, the generation $\overline{T}^* - 2$ would purchase bubbles at her young period, for she could not expect such a consequence occurs at T^*. Therefore in the case of $\omega = 1$ irrational bubbles must collapse at $\overline{T}^* - 1$ when the individuals of the (then old) generation $\overline{T}^* - 2$ tries to sell their bubbles to the (then young) generation $\overline{T}^* - 1$, and accordingly the generation $\overline{T}^* - 2$ incurs the large capital loss. Applying the similar inference, we can easily confirm that the endogenous timing of the crash is $\overline{T}^* - \alpha$ if we assume the foresight ability of all agents is $\omega = \alpha$ [5]. Evidently, the higher the foresight ability of each individual, the bubble collapses at an earlier period.

We can see the endogenous timing of the crash $\overline{T}^* - \alpha$ depends on the various parameters of the economy. We can obtain the following results.

(16) $$\frac{\partial T^*}{\partial \varepsilon} < 0, \quad \frac{\partial T^*}{\partial x_1} < 0 , \quad \frac{\partial T^*}{\partial x_2} > 0 , \quad (\text{ where } x_1 \equiv 1 + \bar{r} \qquad x_2 \equiv s(\bar{r}) a F_2 L \ )$$

From (16) we can see that the larger size of an initial irrational bubbles ε leads to the earlier crash. On the other hand, an increase in the individual savings rate s [6], the labor productivity a, and the

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[5] As is clear from the above discussion, irrational bubbles can not exist if $\omega \geq \overline{T}^*$ holds. This means that the assumption of perfect foresight is not always necessary to eliminate the irrational bubble provided that the foresight ability of agents is assumed to be sufficiently high.

[6] Here an increase in s means one caused by another factors except for a change in the interest rate.

population size L prolongs the duration of the bubble economy. Concerning the interest rate $\bar{r}$, if $\frac{\partial s(\bar{r})}{\partial \bar{r}}$ is negative or a sufficiently small positive value an increase in $\bar{r}$ brings forward the timing of the crash. It is notable that a bubble economy tends to be prolonged in the case where the potential economic growth rate is high. This result stems from the fact that the economy with higher potential of growth (namely, higher rate of the aggregate savings) can allow the larger amount of non-productive savings such as bubbles and hence the irrational bubble lasts longer [7].

3.2 The Growth and Welfare Effects of Irrational Bubbles

The growth effect of irrational bubbles is demonstrated in Figure 3. In general the growth of the real bubble-capital ratio b crowds out new investment for physical capital and then has the negative impact on capital accumulation, as shown in previous researches in this literature. This is because bubble assets work only as the non-productive savings. In our model, however, the economy recovers the potential growth rate $1+\bar{\kappa}$ instantaneously after the crash of bubbles. From the figure below, we can see that the rate of economic growth decelerates gradually as bubbles expand and recovers instantaneously after bursting bubbles.

[Figure 3 around here]

Next we explore the intergenerational welfare effect of irrational bubbles. We confine our analysis to the case of $\omega = 1$ for simplicity, but the result remains valid in other cases. Suppose that the economy is on the non-bubble equilibrium path at first, but the generation –1 sells ε unit of bubble assets to the generation 0 at the initial period 0 and such an intergenerational trade of bubbles is repeated until bubbles burst. Clearly the dynamic path strays from the non-bubble balanced growth equilibrium into the process of irrational bubbles from the period 0.

Let us investigate the welfare implication of such an irrational bubble. Evidently, the welfare of all the individuals of the generation –1 is improved, because they succeeded in exchanging intrinsically useless paper assets with useful consumption goods and then could consume more than otherwise. Concerning the welfare of the generation 0, we can derive the following indirect utility function.

$$(17) \quad U_0^* = V[\bar{\psi} \times K_0] \quad (\text{ where } \bar{\psi} \text{ is a constant })$$

Obviously the occurrence of irrational bubbles does not affect the welfare of her. This is because (1) in this model a physical capital and a bubble asset is equivalent in the rate of return and (2) the welfare of the generation 0 is not influenced by the negative effect of irrational bubbles on capital accumulation because such a negative effect has not been realized yet at the period 0. Finally, the indirect utility function of the generation s is given by

[7] This result seems to indicate that one of the reasons why bubbles in the Japanese economy persisted for a long period in the latter half of 1980's is that the Japanese economy at that time had the high potential of economic growth.

$$U_s^* = V\left[\overline{\psi} \times K_0 \times \prod_{i=0}^{s-1}(1+\kappa_i)\right] \tag{18}$$

Contrary to the generation 0, the welfare of the future generation s ($s = 1,2,\cdots$) is deeply related with the existence of irrational bubbles. From (18) we can see that the lower rate of economic growth in the future corresponds to the lower utility of the generation s . This is because the marginal product of labor (namely, the wage rate) depends on the level of aggregate capital stock at that time, and a decline in the growth rate harms capital accumulation. Since the expansion of irrational bubbles depresses the growth rate as seen above, the existence of irrational bubbles has the negative effect on the welfare of all the future generations [8] [9] . Here we must add the following two remarks. First, if we assume the foresight ability of each individual is $\omega = 1$, the generation $\overline{T}^* - 2$ suffers much larger welfare loss than the other future generations due to the huge capital loss of bubble assets. In general the process of irrational bubbles causes welfare damage to some future generation much more heavily than another generations through the crash, which is characteristic of the case of irrational bubbles. Second, the welfare of future generations born after the crash is similarly harmed in spite of the fact that the growth rate of the economy at that time has already recover the potential growth rate $1+\overline{\kappa}$. This is because the negative impact of the delay of capital accumulation caused by irrational bubbles before the crash persists permanently.

In summary the existence of irrational bubbles has the negative welfare effect on all the future generations, and only the current old (namely, the generation –1 in this model) enjoys the capital gain which arises when she sells bubble assets the generation 0. Even if this capital gain would be very small, the succeeding future generations could incur the large welfare loss once the growth of irrational bubbles accelerates. This implies that the emergence, expansion, and bursting of irrational bubbles has almost Pareto-worsening welfare impact on the economy.

3.3 Irrational Bubbles and Public Policy

In the previous subsection we have demonstrated that the existence of irrational bubbles harms the welfare of all the future generations and especially does serious damage to some future generation who incurs the huge capital loss. In general, the more long time it takes to come back to the balanced growth path without bubbles, the less desirable it seems from the welfare aspect, because the negative effect of the delay of capital accumulation before the crash persists perpetually. So if we could have irrational bubbles burst at an earlier stage by performing some public policy, such a policy would be helpful to improve the welfare. In this subsection we show that a tax on the sale of bubble assets can be effective

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[8] If the economy is on the non-bubble balanced growth path, the indirect utility function of the generation s is given by $\overline{U}_s^* = V[\overline{\psi} \times K_0 \times (1+\overline{\kappa})^s]$ where $1+\overline{\kappa}$ is the gross rate of equilibrium growth without bubbles. Since $1+\overline{\kappa}$ is larger than that with bubbles at any period, the existence of bubbles harms the welfare of all future generations.

[9] This result is well known in the context of asset bubbles and endogenous growth. See, for example, Grossman and Yanagawa(1993).

for such a purpose.

The model is modified as follows by an introduction of a tax on the sale of bubble assets.

Max $U(c_t^y, c_{t+1}^o) = V[u(c_t^y, c_{t+1}^o)]$

s.t. $c_t^y + [s_t + p_t m_t] = w_t$, $c_{t+1}^o = (1+r_{t+1}) + (1-\tau) p_{t+1} m_t$

Here we suppose that the tax rate remains constant over time. The intertemporal budget constraint of the individual is given by $c_t^y + \dfrac{c_{t+1}^o}{1+r_{t+1}} + \left[p_t - \dfrac{(1-\tau) p_{t+1}}{1+r_{t+1}} \right] \times m_t = w_t$. In equilibrium the following arbitrage equation must be satisfied.

(19) $$1 + r_{t+1} = \frac{(1-\tau) p_{t+1}}{p_t}$$

Accordingly the intertemporal budget constraint of the individual is reduced to (3). The government spending $G_t = \tau \times p_t m_{t-1} L$ is assumed to be a wasteful expenditure in that it is introduced into neither the utility function nor the production function.

Deriving the dynamic system under this setting, we can show that only the equation (11.a) is modified as follows.

(20) $$b_{t+1} = \frac{\tilde{x}_1 b_t}{x_2 - b_t} \quad \left(\text{where } \tilde{x}_1 \equiv \frac{1+\bar{r}}{1-\tau} \right)$$

Hence an increase in the tax rate τ means a rise in parameter $\tilde{x}_1$, which leads to the earlier crash as argued in section 3.1. Of course, this does not mean that irrational bubbles can be prevented by imposing a tax on their sale, but at least imposing a tax can cut the duration of a bubble economy short. This result is due to the fact that more rapid growth of price of bubble assets is needed to satisfy the arbitrage equation (19) by introducing a tax. If another conditions remain unchanged, this brings forward the timing of the crash because the real stock of bubbles exceeds the aggregate savings of the young generation at earlier period.

4. Conclusion

In this paper we formulated the irrational bubble theoretically and investigated the properties of it. First, we showed that once the assumption of perfect foresight is removed dynamic paths that the real bubble-capital ratio grows monotonically over time can not be eliminated at the initial period. If such a case occurs, bubbles must collapse by some future period when the real stock of bubbles exceeds the aggregate savings of the young generation. We examined the endogenous timing of the crash theoretically and demonstrated that a bubble economy tends to be prolonged in the case where the potential economic growth rate is high. We also explored the growth and welfare effects of irrational bubbles. It was shown that even though the initial size of irrational bubbles would be very small it could have negative impacts on capital accumulation and economic growth in the process of expansion and as a result harm the welfare of all the future generations, some generation of whom suffers the serious damage of economic welfare due to the huge capital loss. Finally, we argue the relation between the duration of a bubble economy and a tax policy and showed that an increase in a tax rate on the sale of bubble assets brings forward the timing of the crash.

Our attention in this paper is restricted to the case where a bubble arises in an intrinsically useless paper asset that has a zero market fundamentals, but in reality it arises in the prices of productive assets like stocks or lands. Accordingly, it seems to be interesting to introduce stocks or lands to investigate the economic effects of the more actual experiences of asset price bubbles.

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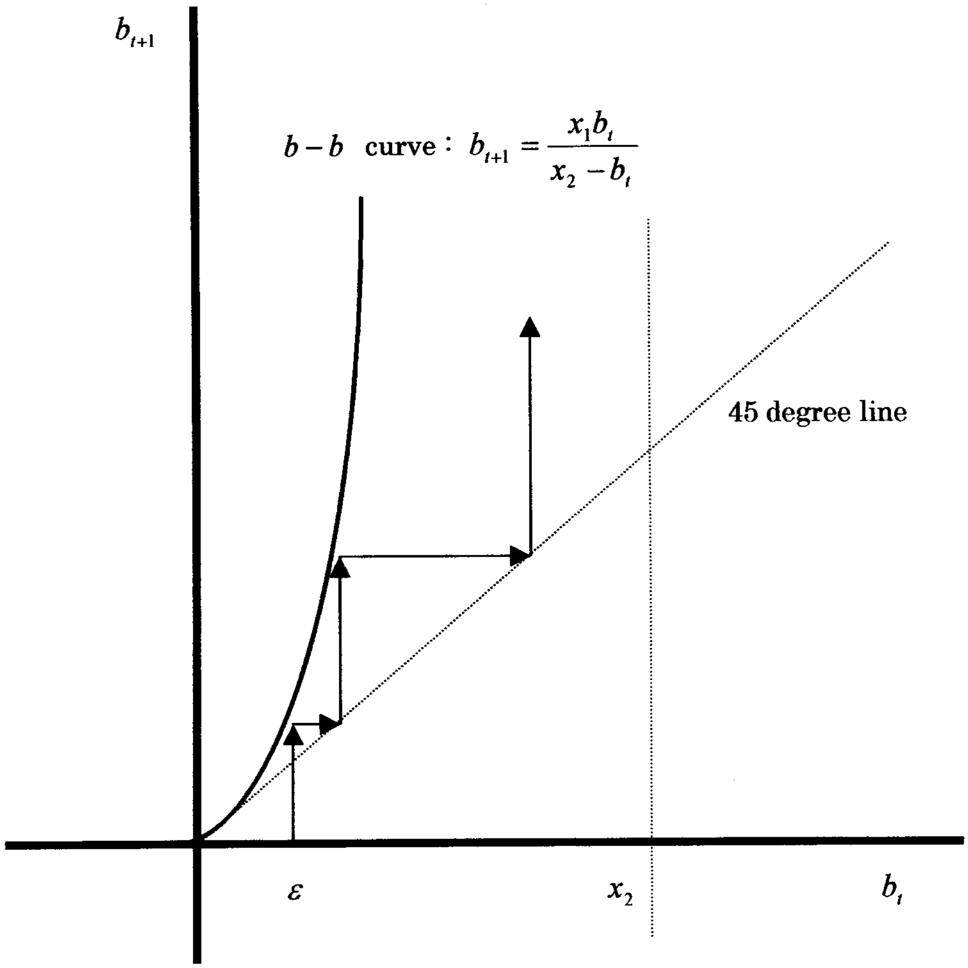

Figure 1 : The phase diagram of (11.a)

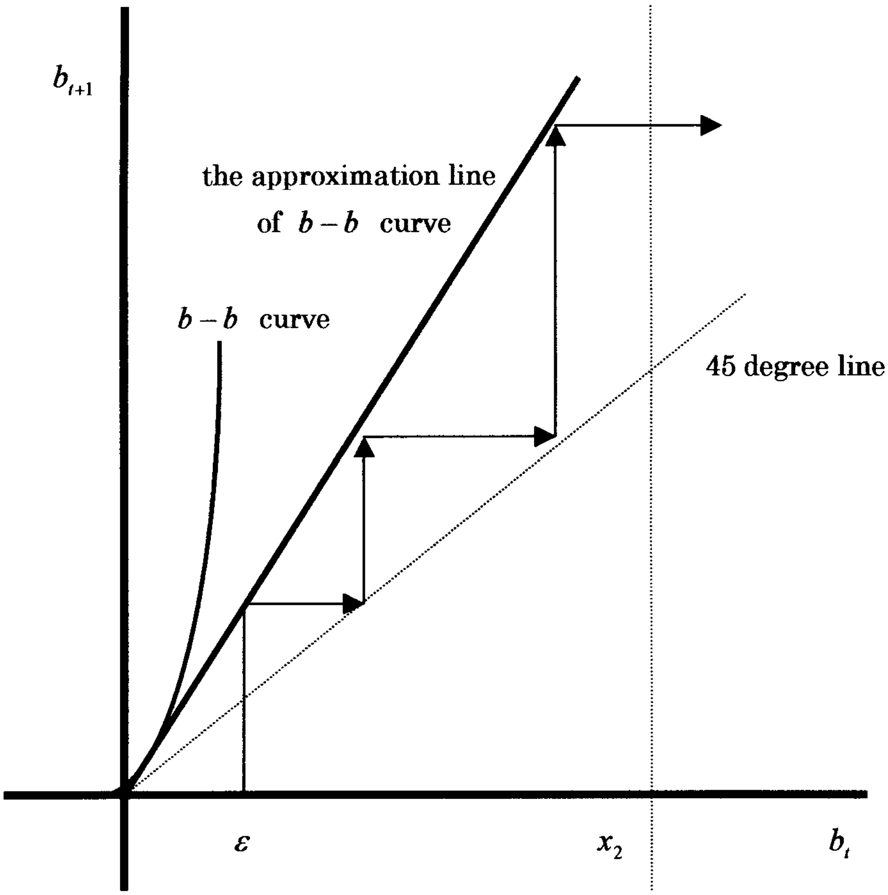

Figure 2 : The approximation line of $b-b$ curve

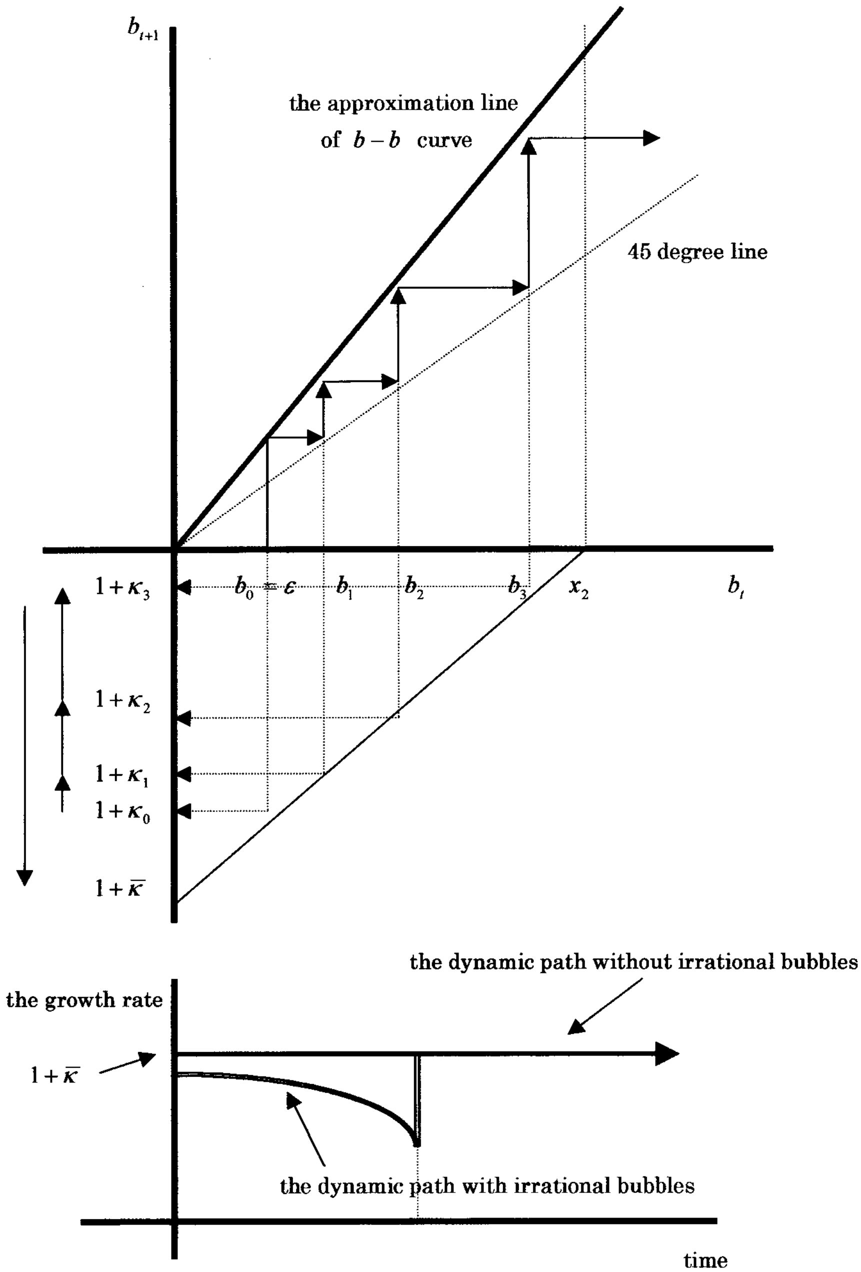

Figure 3 : The relation between irrational bubbles and economic growth