Optimal production of public services in an endogenous growth model*

Noritaka Maebayashi †

Abstract

This study investigates (i) the growth-maximizing, (ii) second-best, and (iii) first-best welfare-maximizing policies when the government produces productive public services using capital and private goods. When the production costs of public services are financed by income tax, the government spending–output ratio exceeds the output elasticity of public services (Pareto-optimal level) under the growth-maximizing or second-best welfare-maximizing policy. This leads to an over-use of resources by the government, over-provision of public services, and under-accumulation of capital in a decentralized economy. Subsidies on private investment, which is increasing in the contribution of publicly employed capital to final goods, can resolve these inefficiencies.

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1 Introduction

The relationship between public services and economic growth has been examined by many economists. In his seminal empirical studies, Aschauer (1988, 1989) finds that the productivity of services from public infrastructure and from investment in public infrastructure in the United States are remarkably high. Based on these findings, Barro (1990) and Futagami, Morita, and Shibata (1993) construct endogenous growth models in which productive public services have positive external effects on output. Furthermore, they show that productive public services can work as a driving force for the sustained growth of an economy. A more important implication of the findings of Barro (1990) and Futagami et al. (1993) is that the growth-maximizing government spending–output ratio is the output elasticity of public services when public spending is financed by a proportional income tax.

This growth-maximizing rule in Barro (1990) and Futagami et al. (1993) has been reconsidered in several extensions to these works. Many theoretical studies indicate that even with such

*Any errors are my responsibility.

†Faculty of Economics and Business Administration, The University of Kitakyushu, 4-2-1 Kitagata, Kokura Minami-ku, Kitakyushu, Fukuoka 802-8577, Japan; e-mail: non818mn@kitakyu-u.ac.jp.

extensions, the growth-maximizing rule in Barro (1990) and Futagami et al. (1993) is strong.[1] From an empirical point of view, the output elasticity of infrastructure (or productive public services) has estimated and examined using data from many countries. Recent empirical studies (e.g., Röller and Waverman, 2001; Shioji, 2001; Esfahani and Ramírez, 2003; Kamps, 2006; Bom and Ligthart, 2014) indicate that the output elasticity of infrastructure (or productive public services) lies in the range 0.1–0.2, on average. However, the growth-maximizing rule in Barro (1990) and Furagami (1993), as well as that studied in various other models suffer from an unrealistic assumption that public services are indistinguishable from final goods. In other words, the government does not produce public services, but instead simply purchases final output from the private sector (see Barro (1990, p. 107)).

In a real economy, the government produces productive public services (goods) not only by purchasing goods from private sector but also by employing human and nonhuman capital.[2] This process of producing public services is supported by the OECD (2017), indicating that government expenditure for the production of public goods and services is classified mainly as compensation of employees in the public sector, or as goods and services used and financed by general government (public intermediate consumption). Accordingly, investigating the relationship between resource allocation for the production of public services and economic growth is an important issue.

The first objective of this study is to investigate the growth-maximizing policy (hereafter, the GM policy) when the government produces productive public services (goods) by employing human and nonhuman capital and by purchasing intermediate goods from the private sector. As part of this process, the government must decide (i) how to allocate human and nonhuman capital between the private and public sectors, and (ii) the share of expenditure between the public employment of capital and intermediate consumption in the public sector.

The second objective is to examine two kinds of welfare-maximizing policies under a production process of public services: the second- and first-best welfare-maximizing policy. The second-best policy (hereafter, SB policy) is a welfare-maximizing policy in a decentralized economy. More specifically, the government decides on the (i) resource allocation between the private and public sectors, and (ii) the share of public expenditure within the public sector. In contrast to the SB policy, the first-best welfare-maximizing policy (hereafter, FB policy) is the Pareto-optimal resource allocations in both the private and public sectors. As for welfare-maximizing fiscal policies, Barro (1990) shows the following two important implications. First, the GM policy coincides with the SB policy; that is, the optimal government spending–output ratio in the decentralized economy is equal to the output elasticity of public services. Second, the government spending–output ratio under the FB policy is also equivalent to the output elasticity of public services. In order to check whether Barro's rule holds when the production process of public services is present, we compare three long-run policies: the GM, SB, and FB policies.

In order to study the GM, SB, and FB policies for the production of public services, we construct a simple endogenous growth model in which the government produces productive public services (goods) by employing human and nonhuman capital and by purchasing intermediate

[1] Exceptions to this view include Gomez (2008) and Agénor (2009). Gomez (2008) incorporates productive public services into the Uzawa–Lucus model, and finds that productive public services do not affect economic growth and there is no growth-maximizing rule. Agénor (2009) shows that the government spending–output ratio under a growth-maximizing policy becomes larger than the output elasticity of public services if public spending on the maintenance of public infrastructure decreases the depreciation of private capital.

[2] According to Turnovsky and Pintea (2006), governments invite bids from private contractors, who employ private capital to carry out the project (nonhuman capital), in order to pass legislation on public investment.

goods from the private sector. We deal with pure public goods (non-rival and non-excludable productive government services) and consider balanced budget public finance based only on income tax, in line with Barro (1990) and Futagami et al. (1993). In this study, the government decides the growth- and welfare-maximizing (i) tax rate and (ii) share of tax revenue allocated to expenditure on the public employment of capital and to intermediate consumption.

The main contributions and the policy implications of this study are as follows. First, the GM, SB, and FB policies depend not only on the output elasticity of public services, but also on the production technology of public services. More specifically, the contribution of resources to the production of public services is a crucial determinant of the optimal production of public services.

Second, the government spending–output ratio under the GM policy becomes larger than the output elasticity of public services, in contrast to the findings of Barro (1990) and Futagami et al. (1993). Furthermore, the outcomes under the GM, SB, and FB policies differ from each other, which breaks Barro's rule. Both the government spending–output ratio and the share of expenditure on the public employment of capital under the GM policy are larger than those under the SB policy. In addition, the ratio and share of expenditure under the FB policy are lower than those under the GM and SB policies.

Finally, the government spending–output ratio under the FB policy is equal to the output elasticity of public services, as in Barro (1990), even when the production process of public services is present. In contrast to Barro (1990), this optimal size of government cannot be realized in a decentralized economy when the production costs of public services are financed by income tax. That is, neither the GM nor the SB policy can attain the Pareto-optimal production of public services. In order to achieve the Pareto optimal allocation of resources, an additional policy instrument is necessary. Here, we find that a subsidy on investment in human and nonhuman capital is an appropriate instrument. We show that the optimal subsidy rate is increasing in the output elasticity of capital employed in the public sector. Without the additional policy instrument, there is an over-employment of capital by the government, over-provision of public goods relative to the size of the economy, and under-accumulation of capital. These differences from Pareto optimality are increasing in the contribution of publicly employed capital to the production of public services.

The differences from the Pareto-optimal production of public services are attributed to the interaction between distortionary income tax and the market rental price of capital that the government employs. Distortionary income tax leads to lower growth of capital than its efficient level. Accordingly, the government in a decentralized economy increases its employment of capital and attempts to raise the market price in order to foster growth of capital. An increase in the rental price of capital has a negative effect on public employment. However, the long-run benefits of economic growth, driven by an increase in the public employment of capital, are stronger. This leads to an over-employment of capital, a higher government spending–output ratio, an over-provision of public goods relative to the size of the economy, and an under-accumulation of capital.

Related Literature

There are a few theoretical studies that examine the public production of government services in the economic growth literature (e.g., Dasgupta, 1999; Turnovsky and Pintea, 2006). However, these studies differ from the present study in the following respects.

First, Dasgupta (1999) focuses on the case of impure public goods, with excludability. With excludability, the government can charge the private sector for the use of public services at the market price. Thus, a zero income tax rate is realized under both the GM and the SB policies. However, as in Dasgupta (1999), we find an over-employment of capital by the government. Therefore, the possibility of an over-provision of public services would be independent of whether or not the public services are non-excludable. In spite of this similarity, we show that the mechanism behind the over-use of resources by the government in our study is different to that of Dasgupta (1999).

Second, Turnovsky and Pintea (2006) use the neoclassical growth model and assume that the government decides on public employment by its instantaneous cost minimization. Therefore, they do not consider long-run optimal public policies. This is not in line with the long-term strategy of the resource management in the public sector for economic growth proposed by the OECD (2011). Furthermore, Turnovsky and Pintea (2006) show there is an under-employment of labor and physical capital in the public sector, and an under-provision of public capital relative to output, both of which differ from the results of this study.

Furthermore, our study is related to the literature on the optimal allocation of government spending. Several recent studies have investigated the problem of allocating expenditure between government infrastructure services and other types of government expenditure. For example, Rioja (2003), Kalaitzidakis and Kalyvitis (2004), and Agénor (2009) consider expenditure on the maintenance of public capital. Then, Agénor (2008) examines public health spending, and Maebayashi (2013) studies social security spending. Kafkalas, Kalaitzidakis, and Tzouvelekas (2014) consider the monitoring of expenditure to detect tax evasion. However, the resource-allocation problems within the public sector examined in this literature are different from those associated with the public production process in our study.

Finally, in the theoretical literature of economic growth and public spending, the findings on the issue of whether the size of the government relative to GDP is efficient or inefficient are inconclusive. Barro (1990) shows that the public policy in a decentralized economy leads to an efficient size of government. Ghosh and Roy (2004) show that the optimal government spending–output ratio in a decentralized economy is lower than that in a centrally planned economy, using an endogenous growth model with a flow of public services and a stock of public capital. Dasgupta (1999) indicates the possibility of over-employment of capital by the government. Here, empirical evidence is again inconclusive. For example, Karras (1996, 1997) shows that the government sizes in OECD countries and in most European countries are efficient. However, Bom and Ligthart (2014) show that public infrastructure is undersupplied in OECD economies. On the other hand, Tanzi and Schuknecht (2000) conclude that the government sizes in the high-income European economies exceeded the optimal level in the second half of the 20th century. Facchini and Melki (2013) also show that the government size in France has been continuously larger than the efficient level since the 1950s. Our results and those in Dasgupta (1999) are partly consistent with the empirical findings of Tanzi and Schuknecht (2000) and Facchini and Melki (2013).

The remainder of this paper proceeds as follows. Section 2 introduces the model. Section 3 characterizes the equilibrium of the market economy. Section 4 examines long-run government policies in market equilibrium. Section 5 solves the optimization problem of a centrally planned economy, and characterizes the Pareto-optimal production of public services. Section 6 assesses the inefficiency caused by market failures in a decentralized economy when the public services are financed by income tax, and proposes public policies that attain Pareto-optimal production in a decentralized economy. Finally, Section 7 concludes the paper.

2 Basic Framework

Consider an economy composed of an infinitely lived representative household (the population size is unity), a continuum of competitive firms, and the government. One kind of traded good is produced by the firms and these goods can be used for consumption or investment. Capital in this economy is viewed broadly as encompassing both human and nonhuman capital, as in Barro (1990). The government balances its budget at each moment in time by levying a flat tax rate on output. It purchases goods and employs capital from the private sector to produce productive government services. Because these services are pure public goods, they are used free of charge for both private and public activities.

2.1 Production of Final Goods

We assume that each firm is identical and they sum to unity. In each firm, the final good's production technology is assumed to take a Cobb–Douglas form,

$$Y_t = AS_{g,t}^{\alpha}[(1-u_{g,t})K_t]^{1-\alpha}, \qquad A>0, \quad 0<\alpha<1, \tag{1}$$

where Y_t and K_t are the final output and (human and nonhuman) capital, respectively,[3] $S_{g,t}$ represents the productive government services,[4] and $u_{g,t}$ is the public employment ratio, that is, the ratio of capital allocated to the public sector in period t. Each firm takes $S_{g,t}$ and $u_{g,t}$ as given. Competitive firms decide on their capital input, $(1-u_{g,t})K_t$, so that the marginal cost of capital is equal to the marginal product. Therefore, the rental rate of capital, r_t, is given by

$$r_t = A(1-\alpha)S_{g,t}^{\alpha}[(1-u_{g,t})K_t]^{-\alpha} = \frac{1-\alpha}{1-u_{g,t}}\left(\frac{Y_t}{K_t}\right). \tag{2}$$

Here, r_t encompasses the rate of return from both human and nonhuman capital because K_t includes both type of capital. Equation (2) indicates that a rise in the public employment ratio, $u_{g,t}$, increases the marginal product of capital in the private sector. Hereafter, we refer to this as the *capital-intensity effect*.

The specification of the production function in (1) assumes constant returns to scale at the social level, but decreasing returns to scale at the private level. This allows firms to earn positive profits,

$$\pi_t = \alpha Y_t. \tag{3}$$

Because the household owns all shares of the firms, these profits are distributed to it.

2.2 Production of Government Services

The government produces public services, $S_{g,t}$. The production technology of government services is assumed to be

$$S_{g,t} = G_t^{\beta_1} S_{g,t}^{\beta_2}(u_{g,t}K_t)^{1-\beta_1-\beta_2}, \quad 0\le\beta_1\le 1, \quad 0\le\beta_2\le 1-\beta_1. \tag{4}$$

[3]Because each firm is identical and they sum to unity, Y_t and K_t are identical to the aggregate output and capital, respectively.

[4]We do not investigate the congestion effect studied by Barro and Sala-i-Martin (1992) and Turnovsky (1997); these are left to future work.

This specification indicates that productive government services are produced by final goods purchased by the government, G_t, and the capital employed in the public sector, $u_{g,t}K_t$. We assume that the government pays the same rental rates, r_t, as in the private sector in each period.[5] As in the case of private goods production, the production process of productive government services, $S_{g,t}$, results in a positive externality. Then, (4) is rewritten as

$$S_{g,t} = G_t^{\omega}(u_{g,t}K_t)^{1-\omega}, \qquad \omega \equiv \frac{\beta_1}{1-\beta_2}. \tag{5}$$

Here, $0 \leq \beta_1 \leq 1$ and $0 \leq \beta_2 \leq 1 - \beta_1$ ensure that $0 \leq \omega \leq 1$. When $\omega = 1$ (i.e., $(\beta_1, \beta_2) = (1-\alpha, \alpha)$ or $(\beta_1, \beta_2) = (1, 0)$), the case of Barro (1990), $S_{g,t} = G_t$, is realized. Thus, (5) includes a more general setting in the production of public services than that of Barro (1990).

2.3 Household

The representative household derives the following utility from the consumption of goods, C_t:

$$U_0 = \int_0^{\infty} e^{-\rho t}\frac{C_t^{1-\sigma}-1}{1-\sigma}dt,$$

where $\rho(>0)$ denotes the subjective discount rate, and $1/\sigma$ is the intertemporal elasticity of substitution. Furthermore, he/she supplies K_t units of capital to the private firms or to the public sector and earns the same income from capital, regardless of whether it is supplied to the private or public sector (see Subsection 2.1). Indifference between supplying capital to the private and public sectors and $K_t = (1-u_{g,t})K_t + u_{g,t}K_t$ leads to the following flow budget constraint for the household:

$$\dot{K}_t = (1-\tau)[r_t(1-u_{g,t})K_t + r_t u_{g,t}K_t + \pi_t] - C_t, \tag{6}$$

where $\tau \in (0,1)$ is the constant income tax rate. The household chooses C_t and $\dot{K}_t$ to maximize its utility subject to (6), taking τ, r_t, and K_0 as given. The first-order conditions yield

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\sigma}[(1-\tau)r_t - \rho], \tag{7}$$

and the usual transversality condition (TVC), $\lim_{t\to\infty} e^{-\rho t}C_t^{-\sigma}K_t = 0$.

2.4 Government

For the production of pure public services, the government levies a flat tax on income at rate τ, and keeps a balanced budget at each moment in time. The tax revenue from households is $\tau[r_t(1-u_{g,t})K_t + r_t u_{g,t}K_t + \pi_t]$, which is equal to $\tau(Y_t + r_t u_{g,t}K_t)$, from (2) and (3). This tax revenue is allocated between spending on final goods purchases and payment for the public employment of capital. Letting E_g denote total government spending, we obtain

$$\tau(Y_t + r_t u_{g,t}K_t) = E_{g,t} = G_t + r_t u_{g,t}K_t. \tag{8}$$

[5]This assumption follows Turnovsky and Pintea (2006).

Let $\phi \in [0, 1]$ be a constant fraction of tax revenue spent on the public employment of capital. Then, (8) leads to

$$\phi\tau(Y_t + r_t u_{g,t} K_t) = r_t u_{g,t} K_t, \tag{9}$$

$$(1-\phi)\tau(Y_t + r_t u_{g,t} K_t) = G_t. \tag{10}$$

As examined in Section 4, the government chooses the policy set (τ, ϕ) to satisfy its long-run objectives; namely, growth or welfare maximization. For these long-run objectives, the government must consider resource allocations both *within* the public sector and *between* the private and public sectors. The former represents the allocation of tax revenue between the cost of public employment, $r_t u_{g,t} K_t$, and final goods purchases, G_t. The latter represents the choices of public employment, $u_g K_t$, which increases public services, but crowds out private inputs, $(1 - u_{g,t})K_t$. These government policies are different from that in Turnovsky and Pintea (2006), in which the government minimizes its instantaneous costs for the production of public goods.

3 Market Equilibrium

In this section, we derive the market equilibrium and characterize the dynamics of the economy. Substituting (5) into (1), we can rewrite the final good's production technology as

$$Y_t = A[(1 - u_{g,t})K_t]^{1-\alpha}(u_{g,t}K_t)^{\alpha(1-\omega)} G_t^{\alpha\omega}. \tag{11}$$

Here, we find that final goods purchases, G_t, and the public employment of capital, $u_{g,t}K_t$, have positive externalities on final output, Y_t, at rates of $\alpha\omega$ and $\alpha(1-\omega)$, respectively.

Substituting (2) into (9) and (10) yields the public employment ratio, u_g, and the public purchase ratio, G_t/Y_t, respectively:

$$u_g = \frac{\phi\tau}{1 - \alpha(1 - \phi\tau)}, \tag{12}$$

and

$$\frac{G_t}{Y_t} = \frac{(1 - \alpha u_g)(1-\phi)\tau}{1 - u_g} = \frac{(1-\phi)\tau}{1 - \phi\tau}. \tag{13}$$

(12) and (13) state the following. First, both the public employment ratio, u_g, and the public purchase ratio, G_t/Y_t, become constant over time. Second, an increase in τ has positive effects on both u_g and G_t/Y_t.[6] Third, the choice of ϕ leads to the trade-off in the allocation of public resources *within* the public sector, which is represented by (12) and (13) as $\partial u_g/\partial\phi > 0$ and $\partial(G_t/Y_t)/\partial\phi < 0$, respectively.[7]

The ratio of government spending to output ratio and that of public services to output are derived by substituting (11), (2), (12), and (13) into (8), and by using (5) and (11) as

$$\frac{E_{g,t}}{Y_t} = \frac{\tau}{1 - \phi\tau}. \tag{14}$$

[6] $\frac{\partial u_g}{\partial \tau} = \frac{(1-\alpha)\phi}{[1-\alpha(1-\phi\tau)]^2} > 0$ and $\frac{\partial (G/Y)}{\partial \tau} = \frac{1-\phi}{(1-\phi\tau)^2} > 0$.

[7] $\frac{\partial u_g}{\partial \phi} = \frac{(1-\alpha)\tau}{[1-\alpha(1-\phi\tau)]^2} > 0$ and $\frac{\partial (G/Y)}{\partial \phi} = -\frac{(1-\tau)\tau}{(1-\phi\tau)^2} < 0$.

and

$$\frac{S_{g,t}}{Y_t} = u_g^{1-\omega}\left(\frac{G_t}{Y_t}\right)^{\omega}\left(\frac{K_t}{Y_t}\right)^{1-\omega} = A^{-\frac{1-\omega}{1-\alpha\omega}}\left(\frac{u_g}{1-u_g}\right)^{\frac{(1-\alpha)(1-\omega)}{1-\alpha\omega}}\left(\frac{G_t}{Y_t}\right)^{\frac{\omega(1-\alpha)}{1-\alpha\omega}}, \tag{15}$$

respectively.

From (2), (3), (6), and (8), the resource constraint of the economy is given by $\dot{K}_t = Y_t - C_t - G_t$. Substituting (11) into it and using (12) and (13), we obtain $\gamma_K \equiv \frac{\dot{K}_t}{K_t} = \frac{1-\tau}{1-\phi\tau}\frac{Y_t}{K_t} - \frac{C_t}{K_t} = \frac{\Psi(1-\tau)\Omega(\tau,\phi)}{1-\alpha(1-\phi\tau)} - \frac{C_t}{K_t}$, where $\Psi \equiv A^{\frac{1}{1-\alpha\omega}}(1-\alpha)^{\frac{1-\alpha}{1-\alpha\omega}}$ and $\Omega(\tau,\phi) \equiv \tau^{\frac{\alpha}{1-\alpha\omega}}(1-\phi)^{\frac{\alpha\omega}{1-\alpha\omega}}\phi^{\frac{\alpha(1-\omega)}{1-\alpha\omega}}(1-\phi\tau)^{-\frac{\alpha}{1-\alpha\omega}}$. Next, substituting (2) and (11) into (7), and using (12) and (13), we obtain $\gamma_C \equiv \frac{\dot{C}_t}{C_t} = \frac{1}{\sigma}[\Psi(1-\tau)\Omega(\tau,\phi) - \rho]$. Here, γ_C is positive as long as $\Psi(1-\tau)\Omega(\tau,\phi) > \rho$. When we define $C_t/K_t \equiv x_t$, we have $\dot{x}_t/x_t = \gamma_C - \gamma_K = x_t + \frac{1}{\sigma}\left[\frac{1-\sigma-\alpha(1-\phi\tau)}{1-\alpha(1-\phi\tau)}\right]\Psi(1-\tau)\Omega(\tau,\phi) - \frac{\rho}{\sigma}$. Thus, x_t, jumps to the following unique steady state and takes a strictly positive value,

$$x = \frac{1}{\sigma}\left[\frac{\sigma-1+\alpha(1-\phi\tau)}{1-\alpha(1-\phi\tau)}\right]\Psi(1-\tau)\Omega(\tau,\phi) + \frac{\rho}{\sigma}, \tag{16}$$

as long as $\frac{1}{\sigma}\left[\frac{\sigma-1+\alpha(1-\phi\tau)}{1-\alpha(1-\phi\tau)}\right]\Psi(1-\tau)\Omega(\tau,\phi) + \frac{\rho}{\sigma} > 0$.[8] To ensure this condition, we assume $\sigma \geq 1$.[9] Because this economy is initially on the balanced-growth path (BGP), in which consumption and the stock of capital grow at the same constant rate $\gamma = \gamma_C = \gamma_K$, we obtain

$$\gamma = \frac{1}{\sigma}[\Psi(1-\tau)\Omega(\tau,\phi) - \rho]. \tag{17}$$

Here, the necessary and sufficient condition that ensures $\gamma > 0$ is

$$\Psi(1-\tau)\Omega(\tau,\phi) > \rho. \tag{18}$$

Because $\dot{C}_t/C_t = \dot{K}_t/K_t = \gamma$ holds from the initial state, consumption and capital stock in period t are expressed as $C_t = C_0e^{\gamma t}$ and $K_t = K_0e^{\gamma t}$, respectively. By substituting these into the TVC, we find that the TVC is satisfied along the BGP if $\gamma(1-\sigma) - \rho < 0$. Obviously, under $\sigma \geq 1$ and (18), the TVC is satisfied. We summarize these results in the following proposition.

Proposition 1

A unique BGP exists in which $\sigma \geq 1$ and (18) ensure $\gamma > 0$, $x > 0$, and the TVC. No transitional dynamics exist in the market economy.

Without the production of public services, $\omega = 1$, this model is identical to that of Barro (1990) and, thus, derives the decentralized equilibrium condition in Barro (1990). Therefore, as would be expected, Barro's (1990) equilibrium conditions are a special case of $\omega = 1$ in this model.[10]

[8] This means that there are no transitional dynamics, as in the standard AK model.

[9] Most empirical results support $\sigma \geq 1$.

[10] If we now reapply the method presented in Section 2, we obtain the decentralized equilibrium condition in Barro (1990), as follows. First, public service is identical to public purchases, $S_{g,t} = G_t$, and the budget constraint of the government is $G_t = \tau Y_t$. Second, the budget constraint of the representative household is $\dot{K}_t = (1-\tau)Y_t - C_t$ and the technology of the final goods sector is $Y_t = AS_{g,t}^{\alpha}K_t^{1-\alpha}$.

4 Government Policies in the Market Economy

We now examine the GM and SB policies. Here, we denote item X as a solution to the growth-maximization and welfare-maximization problems, X^{GM} and X^{SB}, respectively.

4.1 Growth Maximization

From (17), setting $\partial\gamma/\partial\tau = 0$ and $\partial\gamma/\partial\phi = 0$, and solving, yields the following results.

Proposition 2

The income tax rate, expenditure share and government spending–output ratio that maximize the long-run growth rate are given by

$$\tau^{GM} = \alpha, \qquad \phi^{GM} = \frac{1-\omega}{1-\alpha\omega} \quad \text{and} \quad \left(\frac{E_g}{Y}\right)^{GM} = \frac{\alpha(1-\alpha\omega)}{1-\alpha} > \alpha.$$

The public employment ratio, public purchase ratio, ratio of public goods to private goods (measured by GDP), and long-run growth rate under the GM policy are obtained as

$$u_g^{GM} = \frac{(1-\omega)\alpha}{1-\alpha+\alpha(\alpha-\omega)}, \qquad \left(\frac{G}{Y}\right)^{GM} = \alpha\omega,$$

$$\left(\frac{S_g}{Y}\right)^{GM} = \Psi^{-(1-\omega)}\left[\frac{(1-\omega)\alpha}{1-\alpha}\right]^{\frac{(1-\alpha)(1-\omega)}{1-\alpha\omega}}(\alpha\omega)^{\frac{\omega(1-\alpha)}{1-\alpha\omega}},$$

and

$$\gamma^{GM} = \frac{1}{\sigma}\left[\Psi(1-\alpha)^{\frac{1-\alpha}{1-\alpha\omega}}(1-\omega)^{\frac{\alpha(1-\omega)}{1-\alpha\omega}}\alpha^{\frac{\alpha}{1-\alpha\omega}}\omega^{\frac{\alpha\omega}{1-\alpha\omega}} - \rho\right].$$

Proof: See Appendix A.

Proposition 2 provides the following interesting implications. Despite the fact that public services exert a positive external effect in both the private and the public sectors, the tax rate under the GM policy, τ^{GM}, is determined only by the output elasticity of government services, α, as in Barro (1990) and Futagami et al. (1993). In contrast, the share of spending under the GM policy, ϕ^{GM}, depends on the parameters in both the private and public production functions (i.e., α, β_1 and β_2). This reflects the resource allocations within the public sector and between the public and private sectors. As a result, the public employment ratio, u_g^{GM}, the public purchase ratio, $(G/Y)^{GM}$, the ratio of government services $(S_g/Y)^{GM}$, and the long-run growth rate, γ^{GM}, also depend on the specification of technology in both sectors. Finally, the government spending–output ratio under the GM policy becomes larger than the output elasticity of government services, α, in contrast to the findings of Barro (1990) and Futagami et al. (1993). This is attributed to the fact that the government spending–output ratio is larger than the income tax rate (see (14)).

4.2 Welfare Maximization

Next, we investigate the SB policy (i.e., the market optimum). The government's problem here is to choose sets of policy variables (τ, ϕ) to maximize the following indirect utility function:[11]

$$U_0 = \int_0^\infty e^{-\rho t} \frac{(xe^{\gamma t})^{1-\sigma} - 1}{1-\sigma} dt = \frac{(x)^{1-\sigma}}{(1-\sigma)[\rho - \gamma(1-\sigma)]}. \tag{19}$$

Because U_0 depends on γ, maximizing social welfare also depends on the specification of technology in both the private and the public sectors. For this welfare maximization, we set the following assumption on the parameters.

Assumption 1 *We assume parameter sets* $(\alpha, \rho, \sigma, \beta_1, \beta_2, A)$*, which prohibit either (I) tax rates* τ *that satisfy* $\frac{\partial^2 U_0}{\partial \tau^2} > 0$ *at* $\frac{\partial U_0}{\partial \tau} = 0$ *for all* $\phi \in [0,1]$ *or (II) expenditure ratios* ϕ *that satisfy* $\frac{\partial^2 U_0}{\partial \phi^2} > 0$ *at* $\frac{\partial U_0}{\partial \phi} = 0$ *for all* $\tau \in (0,1)$.

Then, Appendix B derives the following proposition.

Proposition 3

(i) *The SB policy* (τ^{SB}, ϕ^{SB}) *exists in the market economy.we obtain* $u_g^{SB} = u_g(\tau^{SB}, \phi^{SB})$, $\left(\frac{G}{Y}\right)^{SB} = \frac{G}{Y}(\tau^{SB}, \phi^{SB})$, $\left(\frac{S_g}{Y}\right)^{SB} = \frac{S_g}{Y}(\tau^{SB}, \phi^{SB})$, $x^{SB} = x(\tau^{SB}, \phi^{SB})$, *and* $\gamma^{SB} = \gamma(\tau^{SB}, \phi^{SB})$. *Assumption 1 ensures the uniqueness of* (τ^{SB}, ϕ^{SB})*, as shown in Figure 1.*

(ii) $\tau^{SB} < \tau^{GM}$, $\phi^{SB} < \phi^{GM}$, $\left(\frac{E_g}{Y}\right)^{SB} < \left(\frac{E_g}{Y}\right)^{GM}$ *and* $u_g^{SB} < u_g^{GM}$ *hold, whereas the signs of* $\left(\frac{G}{Y}\right)^{SB} - \left(\frac{G}{Y}\right)^{GM}$ *and* $\left(\frac{S_g}{Y}\right)^{SB} - \left(\frac{S_g}{Y}\right)^{GM}$ *are ambiguous.*

Proposition 3 demonstrates the following. First, the tax rate, τ, the expenditure ratio, ϕ, and the government spending–output ratio, E_g/Y, under the SB policy are all smaller than those under the GM policy.[12] Second, the optimal ratio of public employment, u_g, is lower than that under growth maximization, because $\partial u_g/\partial \tau > 0$ and $\partial u_g/\partial \phi > 0$ (see the paragraph following (12) and (13)). As a result, the GM policy does not coincide with the SB policy, which breaks Barro's rule.

The intuitive explanation for these results is as follows. Under the GM policy, it is expected that the capital-intensity effect (see (2)) and the positive external effect of public services influence growth. These effects increase public employment and promote capital accumulation, but at the cost of current consumption. As a result, $x(= C_t/K_t)$ falls below the market optimal level, which induces a higher tax rate and a higher share of expenditure on public employment than those at the market optimal level.

4.3 Numerical Examples

This subsection derives explicit numerical solutions for the public sector under the SB policy because it is difficult to do so analytically. These solutions are then compared to those under the

[11] We use $C_t = C_0 e^{\gamma t} = K_0 x e^{\gamma t}$ to derive (19). In this study, we specify $K_0 = 1$ and note that the condition that utility be bounded ensures that $\rho > \gamma(1-\sigma)$.

[12] $\tau^{GM} > \tau^{SB}$ and $\phi^{GM} > \phi^{SB}$ under public production of public goods are in line with Bhattacharyya (2016) who assumed the household-producer of productive public goods.

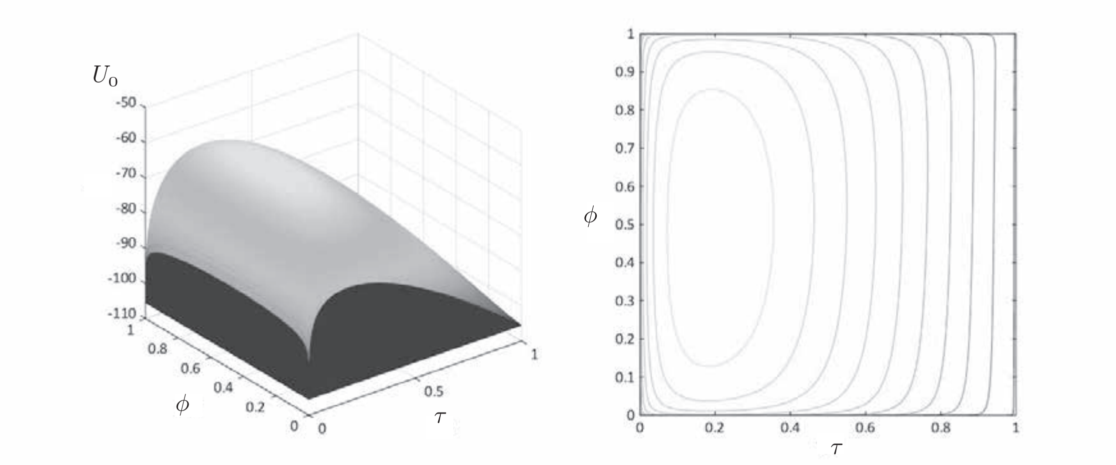

Figure 1: The relationship between U_0 and (τ, ϕ)

Note: $(A, \rho, \sigma, \alpha, \beta_1, \beta_2) = (0.2, 0.04, 1, 0.2, 0.42, 0.2)$.

GM policy. Throughout the subsequent numerical analyses, we assume the following. First, the positive externality of government services is the same for the private and public sectors ($\alpha = \beta_2$). Second, we regard private capital, K_t, as human capital only, and exclude physical capital because there are no certain data on the public employment of physical capital. Furthermore, we set the baseline parameter values as follows:

$$\alpha = 0.2, \quad \beta_1 = 0.42, \quad \beta_2 = 0.2, \quad \sigma = 1, \quad \rho = 0.04, \quad A = 0.2$$

Note that $\sigma = 1$ and $\rho = 0.04$ are widely accepted values. Using the above parameter set, we obtain a plausible long-run growth rate of 0.0264 (0.0263), a ratio of consumption to GDP of 0.6025 (0.6042), a ratio of expenditure for public employment to GDP of 0.1188 (0.0997), and a ratio of expenditure of purchasing goods from the private sector to GDP of 0.1050 (0.1050) under the GM (SB) policy.[13] According to the World Development Indicators, the growth rate of GDP in the OECD countries between 1970 and 2014 was 0.0265, on average, and the ratio of consumption to GDP in the OECD countries between 1970 and 2013 was 0.5977, on average. Furthermore, according to the OECD (2017), the ratio of compensation of general government employees to the average OECD GDP was 10.60%, on average, between 2007 and 2015, while the ratio of public purchases of goods and services to the average OECD GDP was 9.57%, on average, during the same period. Table 1 summarizes the parameter values and compares the steady-state values of the variables, as obtained from model with the data averages.

[13]This fitness is in line with empirical results that developed countries take a growth-maximizing policy (e.g., Karras, 1996; Kamps, 2005; Canning and Pedroni, 2008). Kamps (2005) shows there is no infrastructure shortage in the EU, from the viewpoint of growth maximization. Canning and Pedroni (2008) find that infrastructure is under-provided in some countries and over-provided in others, compared with their respective growth-maximizing levels. Calderón and Servén (2014, p. 10) refer to Kamps (2005) and Canning and Pedroni (2008), and conclude as follows. "On average, the level of infrastructure is 'just about right' from the point of view of growth maximization, so there is no evidence of a generalized infrastructure shortage."

Desciption	GM	SB	Data average
			(the OECD average)
Long-run growth rate of GDP	0.0264	0.0263	0.0265
Consumption-to-GDP ratio	0.6025	0.6042	0.5977
Compensation of government employees-to-GDP ratio	0.1188	0.0997	0.1060
Public intermediate consumption-to-GDP ratio	0.1050	0.1050	0.0957

Table 1: Solutions under GM and SB policies and data averages

Table 2 shows the results for the GM and SB policies under the parameter values given above. The main findings from Table 2 are summarized as follows. $\tau^{SB} < \tau^{GM}$, $\phi^{SB} < \phi^{GM}$, and $u_g^{SB} < u_g^{GM}$ hold, as expected from Proposition 3. In addition, we find that $(\frac{G}{Y})^{SB}$ is approximately equal to $(\frac{G}{Y})^{GM}$, while $(\frac{S_g}{Y})^{SB}$ is smaller than $(\frac{S_g}{Y})^{GM}$, from the numerical example of the average OECD economy. Therefore, both resources used and public services provided by the government under the GM policy are larger than those under the SB policy.

5 Pareto-Optimal Production of Public Services

In this section, we solve the social planner's problem and characterize the Pareto-optimal production of public services. The social planner's objective is to choose C_t, G_t, $u_{g,t}$, and $\dot{K}_t$ to maximize utility, U_0, subject to the resource constraints of the economy. The current value Hamiltonian can be written as $\mathcal{H}_t = \frac{C_t^{1-\sigma}-1}{1-\sigma} + \lambda_t \left\{ A[(1-u_{g,t})K_t]^{1-\alpha}(u_{g,t}K_t)^{\alpha(1-\omega)}G_t^{\alpha\omega} - C_t - G_t \right\}$, where λ_t denotes the co-state variable associated with the resource constraint. From the optimization, we obtain the following solutions:

$$\frac{\partial \mathcal{H}_t}{\partial C_t} = 0 \Rightarrow C_t^{-\sigma} = \lambda_t \tag{20a}$$

$$\frac{\partial \mathcal{H}_t}{\partial u_{g,t}} = 0 \Rightarrow \alpha(1-\omega)(1-u_{g,t}) - (1-\alpha)u_{g,t} = 0 \tag{20b}$$

$$\frac{\partial \mathcal{H}_t}{\partial G_t} = 0 \Rightarrow A\alpha\omega(u_{g,t})^{\alpha(1-\omega)}(1-u_{g,t})^{1-\alpha}G_t^{\alpha\omega-1}K_t^{1-\alpha\omega} = 1 \tag{20c}$$

$$\frac{\partial \mathcal{H}_t}{\partial K_t} = \rho\lambda_t - \dot{\lambda}_t = \lambda_t A(1-\alpha\omega)\left(\frac{Y_t}{K_t}\right), \tag{20d}$$

	τ	ϕ	E_g/Y	u_g	G/Y	S_g/Y
GM policy	0.2000	0.5307	0.2238	0.1293	0.1050	0.3630
SB policy	0.1861	0.4871	0.2047	0.1108	0.1050	0.3369

Table 2: Comparison between the GM and SB policies

Note: $(A, \rho, \sigma, \alpha, \beta_1, \beta_2) = (0.2, 0.04, 1, 0.2, 0.42, 0.2)$.

and the TVC: $\lim_{t\to\infty} e^{-\rho t} C_t^{-\sigma} K_t = 0$. The conditions (20a) to (20d) and (11), together with the initial condition K_0 and the TVC reduce to the dynamics of $x_t (\equiv C_t/K_t)$. The centralized economy jumps to the steady-state, initially, as the market economy does. Along the optimal path, Y_t, K_t, G_t, and C_t all grow at the same constant rate.

By letting item X of the solutions be X^{FB} under the FB policy, we obtain the following proposition, from (20a) to (20d) together with (11) and (15).

Proposition 4

Under the first-best allocation of resources, the public employment ratio, the public purchase ratio, and the ratio of public goods to private goods (measured by GDP) are given by

$$u_g^{FB} = \frac{(1-\omega)\alpha}{1-\alpha+(1-\omega)\alpha}, \qquad \left(\frac{G}{Y}\right)^{FB} = \alpha\omega,$$

and

$$\left(\frac{S_g}{Y}\right)^{FB} = \Psi^{-(1-\omega)}[(1-\omega)\alpha]^{\frac{(1-\alpha)(1-\omega)}{1-\alpha\omega}}(\alpha\omega)^{\frac{\omega(1-\alpha)}{1-\alpha\omega}}.$$

In addition, the economy attains the following long-run growth rate:

$$\gamma^{FB} = \frac{1}{\sigma}\left[\Psi(1-\omega)^{\frac{\alpha(1-\omega)}{1-\alpha\omega}}\alpha^{\frac{\alpha}{1-\alpha\omega}}\omega^{\frac{\alpha\omega}{1-\alpha\omega}} - \rho\right],$$

and the following constant value of x:

$$x^{FB} = \sigma^{-1}(\sigma-1)\Psi(1-\omega)^{\frac{\alpha(1-\omega)}{1-\alpha\omega}}\alpha^{\frac{\alpha}{1-\alpha\omega}}\omega^{\frac{\alpha\omega}{1-\alpha\omega}} + \frac{\rho}{\sigma}.$$

We find that the set of first-best outcomes, (u_g^{FB}, $(G/Y)^{FB}$, $(S_g/Y)^{FB}$, γ^{FB}, x^{FB}) is related to the specification of the production technology in both sectors. This reflects the efficient allocation of resources by the central planner both within the public sector and between the public and private sectors.

As in Barro (1990), we consider whether the FB policy can be implemented under a lump-sum tax finance in a decentralized economy. Under a lump-sum tax finance, the private marginal return on capital is r_t (see (2)), rather than $(1-\tau)r_t$ and, therefore, the Euler equation is given by

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\sigma}\left[\frac{(1-\alpha)}{(1-u_g)}\left(\frac{Y_t}{K_t}\right) - \rho\right] \equiv \gamma_L. \tag{21}$$

Suppose that the lump-sum tax, $T_{L,t}$, is proportional to the total income of households. Then, the tax revenue is given by $T_{L,t} = \tau_L(Y_t + r_t u_{g,t} K_t)$, where $\tau_L \in (0,1)$ is the lump-sum tax rate. The budget constraints of the government are $\tau_L(Y_t + r_t u_{g,t} K_t) = E_{g,t} = G_t + r_t u_{g,t} K_t$, $\phi\tau_L(Y_t + r_t u_{g,t} K_t) = r_t u_{g,t} K_t$, and $(1-\phi)\tau_L(Y_t + r_t u_{g,t} K_t) = G_t$. As in the derivations of (12) and (13), we obtain

$$u_g = \frac{\phi\tau_L}{1-\alpha(1-\phi\tau_L)}, \tag{22}$$

and

$$\frac{G_t}{Y_t} = \frac{(1-\alpha u_g)(1-\phi)\tau_L}{1-u_g} = \frac{(1-\phi)\tau_L}{1-\phi\tau_L}. \tag{23}$$

The optimal policy is to choose (τ_L, ϕ) such that $u_g = u_g^{FB}$, $(G/Y) = (G/Y)^{FB}$ and $\gamma_L = \gamma^{FB}$ hold. From (22), (23), and (21) in the market economy, and $u_g^{FB} = \frac{(1-\omega)\alpha}{1-\alpha+(1-\omega)\alpha}$, $\left(\frac{G}{Y}\right)^{FB} = \alpha\omega$, and $\gamma^{FB} = \frac{1}{\sigma}[(1-\alpha\omega)\frac{Y_t}{K_t} - \rho]$ in the centrally planned economy, we obtain the following result.

Proposition 5

The optimal lump-sum tax rate, optimal expenditure share, and optimal government spending–output ratio under lump-sum tax finance are obtained as

$$\tau_L^{FB} = \frac{\alpha}{1+(1-\omega)\alpha}, \quad \phi^{FB} = 1-\omega, \quad \text{and} \quad \left(\frac{E_g}{Y}\right)^{FB} = \alpha,$$

respectively.

Proposition 5 demonstrates that the Pareto-optimal government spending–output ratio is the output elasticity of public services, α, and the Pareto-optimal share of public expenditure for public employment of capital, $u_g K$, is $1-\omega$: the elasticity of S_g with respect to $u_g K$ (see (5)).

6 Inefficiencies Under Income Tax Finance

Although the provision of public services using solely a lump-sum tax finance attains the first-best outcome, it is difficult to implement in reality. In fact, the governments in many countries use income tax to provide public services. Under income tax finance, government policies such as GM and SB policies in a market economy can depart from Pareto optimality because of the distortionary effect of income tax. However, according to Barro (1990), even with this distortionary effect, the ratio of public services to output (resource use in the public sector) becomes efficient when the government pursues a GM or SB policy. Here, the objective is to investigate whether the resource allocation in the public sector is efficient under a GM or SB policy in a market economy with the public production of public services. If not, we assess the inefficiency caused by the public production of public services.

First, we compare the outcomes under the GM and FB policies. From Propositions 2 and 4, we obtain the following result.

Proposition 6

$u_g^{GM} > u_g^{FB}$, $(G/Y)^{GM} = (G/Y)^{FB}$, $(E_g/Y)^{GM} > (E_g/Y)^{FB}$, $(S_g/Y)^{GM} > (S_g/Y)^{FB}$, *and* $\gamma^{GM} < \gamma^{FB}$ *hold.*

We find that growth maximization induces over-employment in the public sector, and this over-use of resources by the government causes a higher government spending–output ratio ($(E_g/Y)^{GM} > (E_g/Y)^{FB}$) and the over-provision of public goods relative to private goods ($(S_g/Y)^{GM} > (S_g/Y)^{FB}$). In addition, growth maximization causes the under-accumulation of capital (a lower growth rate than that at the first-best outcome). However, the public purchase ratio under growth maximization, $(G/Y)^{GM}$, becomes efficient, as in Barro (1990).

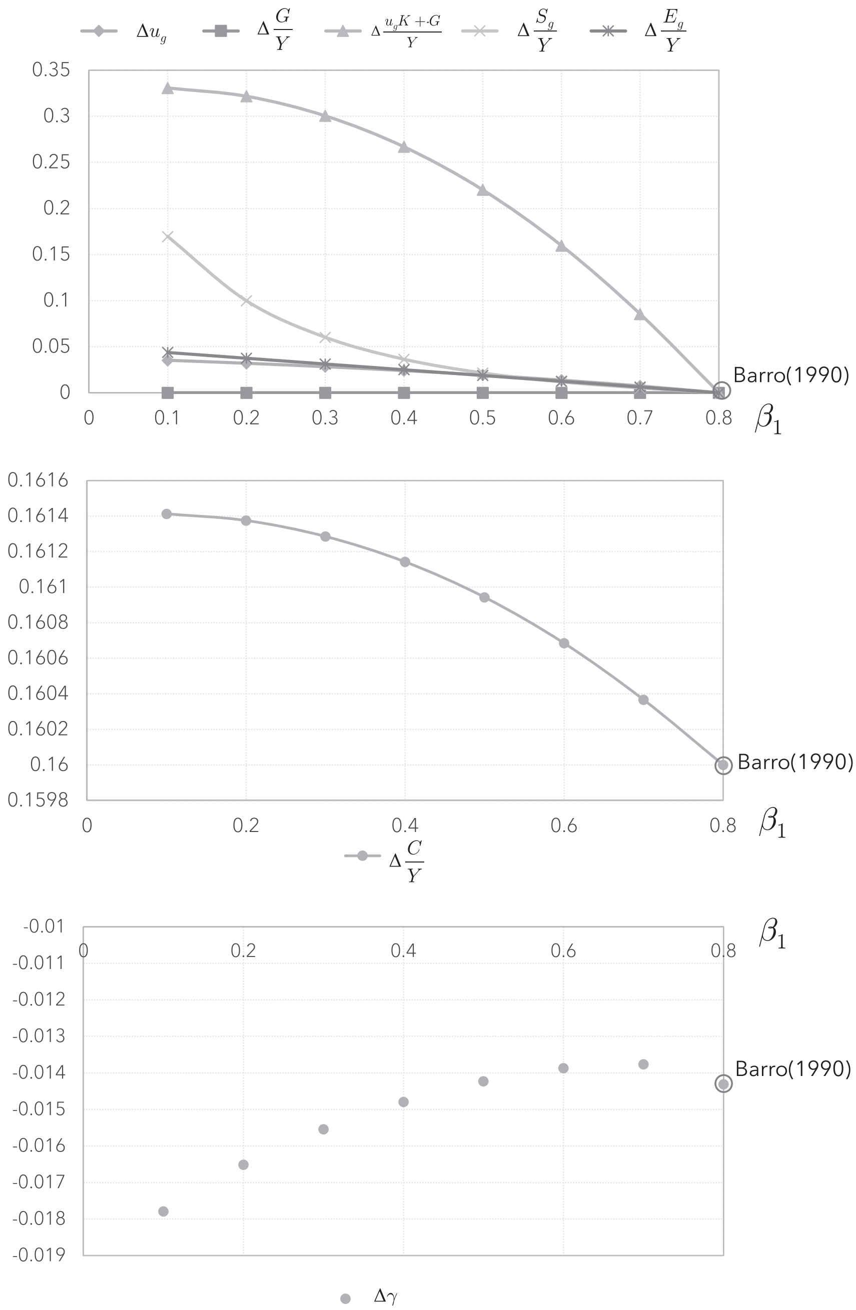

Figure 2: Difference between the FB and GM policies

	E_g/Y	u_g	G/Y	$u_gK/Y+G/Y$	S_g/Y	C/Y	γ
Benchmark case: $\beta_1 = 0.42$							
FB policy	0.2000	0.1061	0.1050	1.2763	0.3302	0.4414	0.0411
SB policy	0.2047	0.1108	0.1050	1.3278	0.3369	0.6042	0.0263
$\beta_1 = 0.2$							
FB policy	0.2000	0.1579	0.0500	1.6061	0.6588	0.3942	0.0564
SB policy	0.2083	0.1652	0.0500	1.6781	0.6815	0.5583	0.0397
$\beta_1 = 0.7$							
FB policy	0.2000	0.0303	0.1750	0.5284	0.1911	0.4665	0.0307
SB policy	0.2011	0.0316	0.1750	0.5441	0.1921	0.6273	0.0170

Table 3: Comparison between the SB and FB policies when $\alpha = \beta_2 = 0.2$

Note $(A, \rho, \sigma) = (0.2, 0.04, 1)$.

	E_g/Y	u_g	G/Y	$u_gK/Y + G/Y$	S_g/Y	C/Y	γ
Benchmark case: $\beta_1 = 0.45$							
FB policy	0.1000	0.0526	0.0500	0.7317	0.1846	0.5181	0.0334
SB policy	0.1007	0.0533	0.0500	0.7403	0.1858	0.6085	0.0264
$\beta_1 = 0.2$							
FB policy	0.1000	0.0795	0.0222	0.9904	0.4185	0.4869	0.0403
SB policy	0.1011	0.0806	0.0222	1.0030	0.4227	0.5775	0.0329
$\beta_1 = 0.7$							
FB policy	0.1000	0.0241	0.0778	0.3998	0.1067	0.5345	0.0290
SB policy	0.1003	0.0244	0.0778	0.4040	0.1070	0.6247	0.0223

Table 4: Comparison between the SB and FB policies when $\alpha = \beta_2 = 0.1$

Note $(A, \rho, \sigma) = (0.124, 0.04, 1)$.

Figure 2 shows the differences in the variables between the GM and FB policies: $\Delta X \equiv X^{GM} - X^{FB}$ when β_1 varies in the range $\beta_1 \in (0, 1-\alpha)$ and $\beta_2 = \alpha$.[14] Then, Δu_g, $\Delta E_g/Y$, $\Delta S_g/Y$, and $\Delta(u_g K + G)/Y$ (the differences in resource use by the government) are decreasing in β_1. Although $\Delta\gamma$ has an inverted-U relationship with β_1, we can understand that $\Delta\gamma$ is also decreasing in β_1, unless β_1 is sufficiently large. From the definition of ω (see (5)), the differences from Pareto optimality tend to be increasing (decreasing) in the contribution of $u_g K$ (G_t) to the production of S_g: $1-\omega$ (ω).

Next, we compare the outcomes under the SB and the FB policies numerically, because it is difficult to do so analytically. We use the same parameter values as in Section 4. Table 2 provides the following results.[15]

Result 1

The SB policy in the market economy causes over-employment in the public sector, $u_g^{SB} > u_g^{FB}$*, an over-use of resources by the government,* $(u_g K/Y)^{SB} + (G/Y)^{SB} > (u_g K/Y)^{FB} + (G/Y)^{FB}$*, a higher government spending–output ratio,* $(E_g/Y)^{SB} > (E_g/Y)^{FB}$*, an over-provision of public goods relative to private goods,* $(S_g/Y)^{SB} > (S_g/Y)^{FB}$*, and an under-accumulation of capital,* $\gamma^{SB} < \gamma^{FB}$*, all of which are common to the growth-maximizing policy.*

According to recent empirical studies (e.g., Röller and Waverman, 2001; Shioji, 2001; Esfahani and Ramírez, 2003; Bom and Ligthart, 2014), the output elasticity of infrastructure lies in the range 0.1–0.2. Therefore, we check the robustness when $\alpha = 0.1$.[16] Table 3 shows that result 1 is robust, even if $\alpha (= \beta_2)$ changes to 0.1.

Next, we define the Pareto disparity of variable X between the SB and FB policies as $\tilde{\Delta} X \equiv X^{SB} - X^{FB}$. How the Pareto disparity changes when β_1 varies in the range $\beta_1 \in (0, 1-\alpha)$ is represented in Figure 3. Figure 3 shows that $\tilde{\Delta} u_g$, $\tilde{\Delta} E_g/Y$, $\tilde{\Delta} S_g/Y$, and $\tilde{\Delta}(u_g K + G)/Y$ are decreasing in β_1. In addition, $\Delta\gamma$ is decreasing in β_1, unless β_1 is sufficiently large. Therefore, the differences from the Pareto optimal value tend to be increasing (decreasing) in the contribution of $u_g K$ (G_t) to the production of S_g: $1-\omega$ (ω).

We consider the intuitive reasoning behind the above result in order to understand the effects of market failures. First, the result of an under-accumulation of capital is common to the findings of previous studies, such as Barro (1990) and Ghosh and Roy (2004). Owing to the distortionary effect of income tax and the positive externality of public services, the market economy enjoys higher current consumption, at the cost of capital accumulation, than in the case of the centrally planned economy.

Next, this negative distortionary effect on capital accumulation drives the government to increase the growth rate, regardless of whether its objective is growth maximization or welfare maximization. Then, the government increases the public employment ratio, u_g, and raises the rental price of capital, r_t, to influence growth through the capital-intensity effect. This is the *pos-*

[14]When $\beta_1 = 1-\alpha$ and $\beta_2 = \alpha$, the case of Barro (1990) is realized, as mentioned in the paragraph following (5).

[15]Note that $(u_g K/Y) + (G/Y)$ can be larger than one because the ratio of private capital to GDP in most OECD member countries is much larger than one, as per the Database on Capital Stocks in OECD Countries of the Kiel Institute for the World Economy.

[16]We choose $\beta_1 = 0.45$ when $\alpha = 0.1$, which yields a plausible ratio of compensation of government employees to public intermediate consumption of 1.10 (1.01) under the GM (SB) policy, because the average OECD value between 2007 and 2014 is around 1.10.

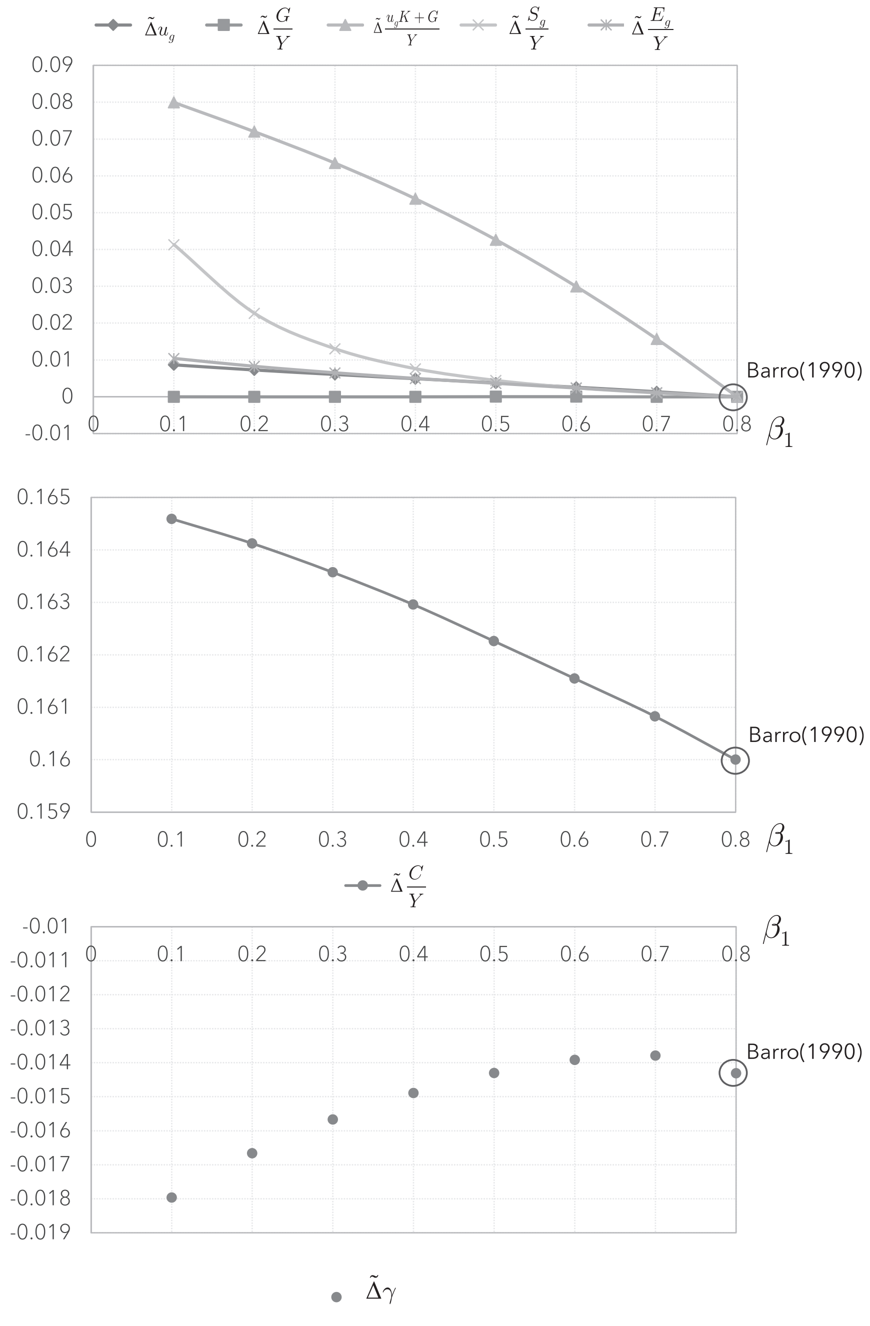

Figure 3: Difference between the FB and SB policies

itive distortionary effect that stems from the production of public services. In contrast, a rise in r_t raises the cost of employing capital and has a *negative distortionary effect* on public employment. The finding of over-employment in the public sector indicates that the positive distortionary effect dominates the negative effect. This over-employment in the public sector leads to the over-use of resources by the government and the over-provision of public goods relative to private goods. The larger the contribution of publicly employed capital to the production of public services, $1-\omega$, the stronger the capital-intensity effect is and, therefore, the difference from the Pareto optimal value increases as $1-\omega$ increases.

Dasgupta (1999) shows that over-employment in the public sector occurs when the government produces services with excludability (i.e., impure public goods). Therefore, over-employment in the public sector occurs independently of whether excludability in public services is present. However, the mechanism of the over-employment in the public sector in Dasgupta (1999) is different from that in this study. In Dasgupta (1999), the government can rely on sales revenue from public, goods owing to their excludability. Hence, the market economy tries to raise welfare by reducing the tax burden to zero. Thus, the market economy can attain a higher growth rate than that of the centrally planned economy. This *over-accumulation of capital* boosts sales revenue of impure public goods and induces the over-employment of capital in the public sector.

Policy Implications

In the rest of this section, we consider the Pareto optimal public policies. In order to achieve Pareto optimality, the government needs to resolve both the over-employment of capital in the public sector and the under-accumulation of capital.

Here, we first introduce the lump-sum tax, T_t, and the consumption tax on the household at rate $\tau_c(>0)$ to finance the subsidy on investment, because these taxes cause no distortion. Thus, we rewrite the household's budget constraint as

$$(1-s)\dot{K}_t = (1-\tau)[r_t(1-u_{g,t})K_t + r_t u_{g,t}K_t + \pi_t] - T_t - (1+\tau_c)C_t, \tag{24}$$

where s denotes the subsidy rate, and $s\dot{K}_t = \tau_c C_t + T_t$ is satisfied. The Euler equation under this constraint is given by

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\sigma}\left[\frac{(1-\tau)(1-\alpha)}{(1-s)(1-u_g)}\left(\frac{Y_t}{K_t}\right) - \rho\right] \equiv \tilde{\gamma}. \tag{25}$$

The optimal policy is to choose (τ, ϕ, s) such that $u_g = u_g^{FB}$, $(G/Y) = (G/Y)^{FB}$ and $\tilde{\gamma} = \gamma^{FB}$ hold. From (12), (13), and (25) in the market economy, and $u_g^{FB} = \frac{(1-\omega)\alpha}{1-\alpha+(1-\omega)\alpha}$, $\left(\frac{G}{Y}\right)^{FB} = \alpha\omega$, and $\gamma^{FB} = \frac{1}{\sigma}[(1-\alpha\omega)\frac{Y_t}{K_t} - \rho]$ in the centrally planned economy, we obtain the following result.

Proposition 7

The government spending–output ratio and expenditure share under the FB policy are consistent with those obtained in Proposition 5: $(E_g/Y)^{FB} = \alpha$ and $\phi^{FB} = 1-\omega$, respectively. The income tax rate under the FB policy , τ^{FB}, also coincides with lump-sum tax rate, τ_L^{FB} in Proposition 5 (i.e., $\tau^{FB} = \frac{\alpha}{1+(1-\omega)\alpha}$). The optimal subsidy rate, s^{FB} is obtained as

$$s^{FB} = \frac{(1-\omega)\alpha}{1+(1-\omega)\alpha}.$$

Proposition 7 indicates that a subsidy on investment in capital can remove inefficiencies. The size of the subsidy is characterized only by the output elasticities of publicly employed capital, $(1-\omega)\alpha$, (see (11)), and is increasing in $(1-\omega)\alpha$. This reflects the aforementioned result that the differences from Pareto optimality are increasing in $1-\omega$.[17]

In addition, from Figure 4, we find that the income tax rate and expenditure share under the FB policy are smaller than those under the GM and SB policies ($\tau^{FB} < \tau^{SB} < \tau^{GM}$ and $\phi^{FB} < \phi^{SB} < \phi^{GM}$). From these, the government spending–output ratio under the FB policy becomes smaller than those under the GM and SB policies ($(E_g/Y)^{FB} < (E_g/Y)^{SB} < (E_g/Y)^{GM}$).

7 Conclusion

We have examined the GM, SB, and FB policies associated with the public production of non-rival and non-excludable productive government services in an endogenous growth model. The optimal policies reflect (i) optimal government spending (taxation) and (ii) an optimal allocation of tax revenue between compensation of publicly employed capital and public intermediate consumption. In this production process of public services, the government must decide on (i) the resource allocation of human and nonhuman capital between the private and public sectors, and (ii) the share of expenditure between the public employment of capital and public intermediate consumption within the public sector. The main results from our investigation are summarized as follows.

First, the GM, SB, and FB policies depend on not only the output elasticity of public services, but also the production technology of public services. More specifically, the contribution of resources to the production of public services is a crucial determinant of the optimal production of public services.

Second, the government spending–output ratio under the GM policy becomes larger than the output elasticity of public services, in contrast to the findings of Barro (1990) and Futagami et al. (1993). Furthermore, the outcomes under the GM, SB, and FB policies differ from each other and, therefore, break Barro's rule. Both the government spending–output ratio and the share of expenditure on the public employment of capital under the GM policy are larger than those under the SB and FB policies.

Finally, the government spending–output ratio under the FB policy is consistent with the output elasticity of public services. However, this optimal size of government cannot be realized in a decentralized economy when the production costs of public services are financed by income tax. In order to achieve the Pareto optimal allocation of resources, a subsidy on investment in capital might be necessary. We show that the optimal subsidy rate is increasing in the output elasticity of capital employed in the public sector. Without such a policy instrument, over-employment of capital by the government, an over-provision of public goods relative to the size of the economy, and an under-accumulation of capital occur. These differences from Pareto optimality are increasing in the elasticity of capital to the production of public services.

[17] From the definition of ω, $(1-\omega)\alpha$ is decreasing in β_1.

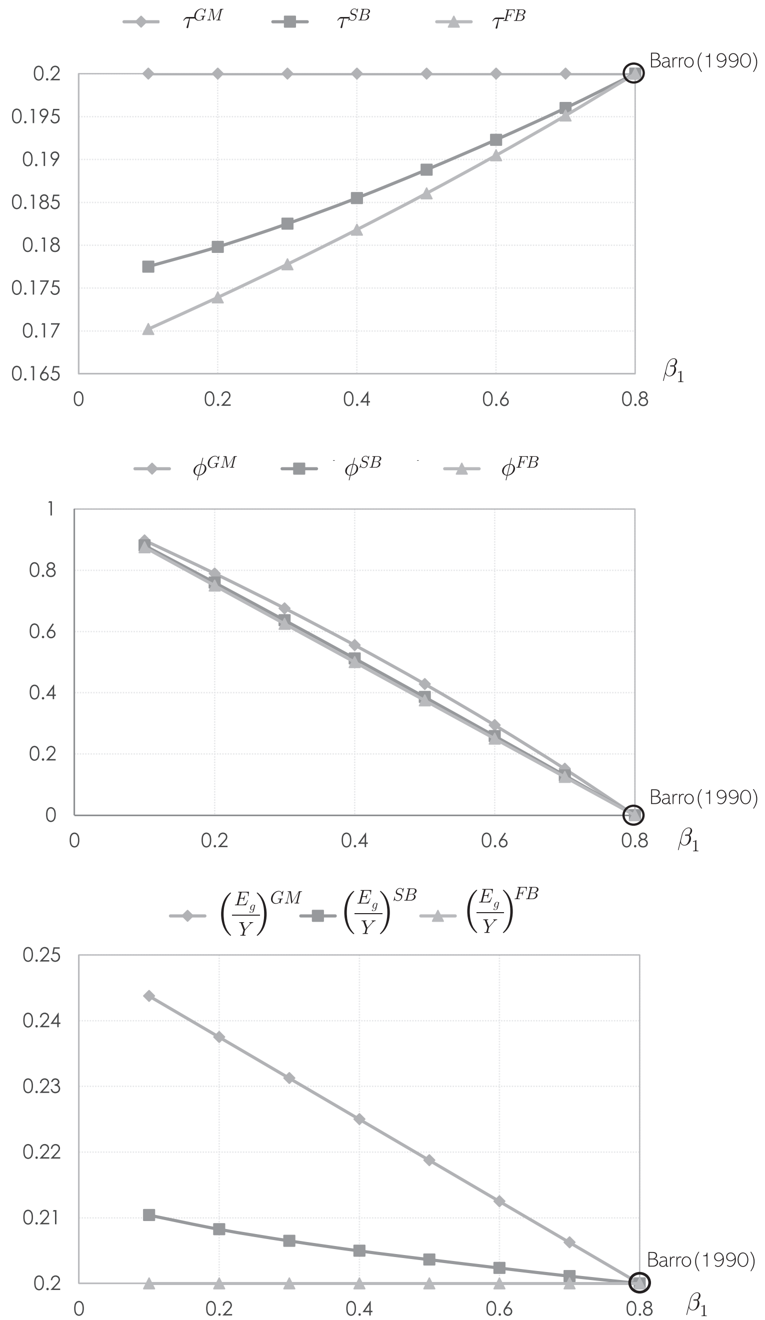

Figure 4: Comparison of policy variables and the government spending–output ratios

Appendix

A Proof of Proposition 2

From (17), maximizing γ with respect to τ and ϕ is equivalent to $\max_{\tau,\phi}\ \ln(1-\tau)\Omega(\tau,\phi)$. The first-order conditions of this problem are

$$\tau: \qquad -\frac{1}{1-\tau}+\frac{\alpha}{1-\alpha\omega}\frac{1}{\tau}+\frac{\alpha}{1-\alpha\omega}\frac{\phi}{1-\phi\tau}=0, \tag{A.1}$$

$$\phi: \qquad -\frac{\omega}{1-\phi}+\frac{1-\omega}{\phi}+\frac{\tau}{1-\phi\tau}=0. \tag{A.2}$$

Rewriting (A.1) as $\frac{\tau}{1-\phi\tau}=\frac{1-\alpha\omega}{\alpha}\frac{\tau}{(1-\tau)\phi}-\frac{1}{\phi}$ and substituting it into (A.2), we obtain

$$\tau=\frac{\alpha\omega}{1-(1-\alpha\omega)\phi}. \tag{A.3}$$

Substituting (A.3) into (A.1), we obtain

$$\phi^{GM}=\frac{1-\omega}{1-\alpha\omega}. \tag{A.4}$$

From (A.3) and (A.4), we obtain $\tau^{GM}=\alpha$. Substituting the values of τ^{GM} and ϕ^{GM} into (12), (13), (14), (15), and (17), we obtain u_g^{GM}, $(\frac{G}{Y})^{GM}$, $(\frac{E_g}{Y})^{GM}$, $(\frac{S_g}{Y})^{GM}$, and γ^{GM}, respectively, in Proposition 2.

B Proof of Proposition 3

By differentiating (19) with respect to τ and ϕ, we obtain

$$\frac{\partial U_0}{\partial\tau}=\frac{x^{1-\sigma}}{[\rho-\gamma(1-\sigma)]^2}\left[\frac{\rho-\gamma(1-\sigma)}{x}\frac{\partial x}{\partial\tau}+\frac{\partial\gamma}{\partial\tau}\right], \tag{B.1}$$

and

$$\frac{\partial U_0}{\partial\phi}=\frac{x^{1-\sigma}}{[\rho-\gamma(1-\sigma)]^2}\left[\frac{\rho-\gamma(1-\sigma)}{x}\frac{\partial x}{\partial\phi}+\frac{\partial\gamma}{\partial\phi}\right], \tag{B.2}$$

where,

$$\begin{pmatrix}\frac{\partial x}{\partial\tau} & \frac{\partial x}{\partial\phi}\\ \frac{\partial\gamma}{\partial\tau} & \frac{\partial\gamma}{\partial\phi}\end{pmatrix}=\begin{pmatrix}-\frac{\alpha\phi(1-\tau)\Psi\Omega(\tau,\phi)}{[1-\alpha(1-\phi\tau)]^2}+\frac{\sigma-1+\alpha(1-\phi\tau)}{1-\alpha(1-\phi\tau)}\frac{\partial\gamma}{\partial\tau} & -\frac{\alpha\tau(1-\tau)\Psi\Omega(\tau,\phi)}{[1-\alpha(1-\phi\tau)]^2}+\frac{\sigma-1+\alpha(1-\phi\tau)}{1-\alpha(1-\phi\tau)}\frac{\partial\gamma}{\partial\phi}\\ \frac{\Psi[-\Omega(\tau,\phi)+(1-\tau)\Omega_\tau(\tau,\phi)]}{\sigma} & \frac{\Psi(1-\tau)\Omega_\phi(\tau,\phi)}{\sigma}\end{pmatrix},$$

$\Omega_\tau(\tau,\phi)=\frac{\alpha\Omega(\tau,\phi)}{(1-\alpha\omega)\tau(1-\phi\tau)}$, and $\Omega_\phi(\tau,\phi)=\frac{\alpha\Omega(\tau,\phi)}{(1-\alpha\omega)}\left[\frac{1-\omega-\phi}{\phi(1-\phi)}+\frac{\tau}{1-\phi\tau}\right]$. From (B.1) and (B.2), growth maximization ($\partial\gamma/\partial\tau=\partial\gamma/\partial\phi=0$) does not coincide with welfare maximization.

Next, we show the existence of the SB policy. Because $\lim_{\tau\to 0}[-\Omega(\tau,\phi)+(1-\tau)\Omega_\tau(\tau,\phi)]=+\infty$ and $\lim_{\tau\to 1}[-\Omega(\tau,\phi)+(1-\tau)\Omega_\tau(\tau,\phi)]=-[\phi/(1-\phi)]^{\frac{(1-\omega)\alpha}{1-\alpha\omega}}$, we obtain $\lim_{\tau\to 0}\partial x/\partial\tau=\lim_{\tau\to 0}\partial\gamma/\partial\tau=+\infty$ and $\lim_{\tau\to 1}\partial x/\partial\tau$, $\lim_{\tau\to 1}\partial\gamma/\partial\tau<0$. Therefore, we obtain

$$\lim_{\tau\to 0}\frac{\partial U_0}{\partial\tau}=+\infty \quad\text{and}\quad \lim_{\tau\to 1}\frac{\partial U_0}{\partial\tau}<0. \tag{B.3}$$

In addition, $\lim_{\phi \to 0} \Omega_\phi(\tau, \phi) = \infty$ and $\lim_{\phi \to 1} \Omega_\phi(\tau, \phi) = -\infty$ lead to

$$\lim_{\phi \to 0} \frac{\partial U_0}{\partial \phi} = +\infty \quad \text{and} \quad \lim_{\phi \to 1} \frac{\partial U_0}{\partial \phi} < 0. \tag{B.4}$$

Because U_0 is a continuous function of τ and ϕ for $\tau \in (0,1)$ and $\phi \in [0,1]$, (B.3) and (B.4) show that there is at least one pair of (τ, ϕ) at which both $\partial U_0/\partial \tau = 0$ and $\partial U_0/\partial \phi = 0$ are satisfied. In other words, there is at least one SB policy.

Finally, we evaluate (B.1) and (B.2) under the GM policy (i.e., $(\tau, \phi) = (\tau^{GM}, \phi^{GM})$ and $\frac{\partial \gamma}{\partial \tau} = \frac{\partial \gamma}{\partial \phi} = 0$). Some easy algebra yields

$$\frac{\partial U_0}{\partial \tau}|_{(\tau,\phi)=(\tau^{GM},\phi^{GM})} = -\frac{\Psi \alpha \phi^{GM}(1-\tau^{GM})\Omega(\tau^{GM}, \phi^{GM})\{x|_{(\tau,\phi)=(\tau^{GM},\phi^{GM})}\}^{-\sigma}}{[\rho - \gamma(1-\sigma)][1-\alpha(1-\phi^{GM}\tau^{GM})]^2} < 0, \tag{B.5}$$

$$\frac{\partial U_0}{\partial \phi}|_{(\tau,\phi)=(\tau^{GM},\phi^{GM})} = -\frac{\Psi \alpha \tau^{GM}(1-\tau^{GM})\Omega(\tau^{GM}, \phi^{GM})\{x|_{(\tau,\phi)=(\tau^{GM},\phi^{GM})}\}^{-\sigma}}{[\rho - \gamma(1-\sigma)][1-\alpha(1-\phi^{GM}\tau^{GM})]^2} < 0. \tag{B.6}$$

From (B.5) and (B.6), $\tau^{SB} < \tau^{GM}$ and $\phi^{SB} < \phi^{GM}$ hold. From (14), $\tau^{SB} < \tau^{GM}$ and $\phi^{SB} < \phi^{GM}$ lead to $\left(\frac{E_g}{Y}\right)^{SB} < \left(\frac{E_g}{Y}\right)^{GM}$. In addition, from the statements in the paragraph following (13) in Section 3, we find that $\frac{\partial u_g}{\partial \tau} > 0$, $\frac{\partial u_g}{\partial \phi} > 0$, and $\phi^{SB} < \phi^{GM}$ indicate that $u_g^{SB} < u_g^{GM}$. Furthermore, $\frac{\partial (G/Y)}{\partial \tau} > 0$ and $\tau^{SB} < \tau^{GM}$ tend to show $\left(\frac{G}{Y}\right)^{SB} < \left(\frac{G}{Y}\right)^{GM}$, while $\frac{\partial (G/Y)}{\partial \phi} < 0$ and $\phi^{SB} < \phi^{GM}$ tend to show $\left(\frac{G}{Y}\right)^{SB} > \left(\frac{G}{Y}\right)^{GM}$. This leads to ambiguity about which is larger, not only between $\left(\frac{G}{Y}\right)^{SB}$ and $\left(\frac{G}{Y}\right)^{GM}$, but also between $\left(\frac{S_g}{Y}\right)^{SB}$ and $\left(\frac{S_g}{Y}\right)^{GM}$.

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