A Note on Strategic Delegation and Trade Policy with Relative Performance

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1 Introduction

Das [2] was the first to highlight the similarity linking strategic trade and managerial delegation. Traditional strategic trade policy analyses, as Brander and Spencer[1] (the BS model hereafter) found that government subsidization induces the domestic firm to behave as the Stackelberg leader under Cournot competition, thus improving its own welfare. Fershtman and Judd [4] and Sklivas [7] (in what we refer to as the FJS model hereafter), identified the equivalent rent-shifting effects in their strategic managerial delegation model. They clarified that delegating a manager with distorted objective functions also encourages the firm to act as a Stackelberg leader under quantity competition.

Das [2] adopted the incentive contract in the FJS model, which is a linear combination of the own firm profit and revenue. Besides the FJS contract, other forms of incentive contracts also yield interesting results. Miller and Pazgal[6] (the MP model hereafter) adopted the relative performance contract, which is a linear combination of the own firm and rival firm profit. They found that the equivalence resulted in the optimal strategic trade policy and market outcomes regardless of whether the firms compete by setting prices or quantities.

In this note, we reexamine the implications of the separation of ownership and management based on relative performance contracts following the MP model. We focus on the nature of the equivalent strategic behavior between government trade policy and managerial delegation under duopolistic competition. We show that the strategic subsidization incentive depends on the presence of managerial delegation. We further endogenize the firms' managerial delegation decisions and find that both firms choose not to delegate a manager, and the socially desirable outcome is the result.

The rest of the paper proceeds as follows. In Section 2, we describe a four-stage game in a third-market setting. In Section 3, we derive the optimal outputs in the last stage when

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both firms choose delegation. In Section 4, we discuss the government subsidy decisions in the four subgames regarding the owners' delegation decisions in the first stage. In Section 5, we endogenize owners' delegation decisions and solve for the game equilibrium. In the las section, we provide our concluding remarks.

2 Model Setup

Following the framework of the MP model, we consider two exporting countries (1 and 2), each with one firm producing a differentiated product. The two countries' products are imperfect substitutes and sell in a third country, the importing country.

We define the aggregate or representative utility functions of the third country in the following quasi-linear functions:

$$U(q_1, q_2; z) = a(q_1 + q_2) - \frac{q_1^2 + q_2^2}{2} - \gamma q_1 q_2 + z,$$

where q_1 and q_2 denote the consumption of imperfect substitute goods supplied by firm 1 and 2, respectively, while z denotes the consumption of a numeraire competitive good. $\gamma \in (0, 1)$ represents the degree of substitutability between the two goods. An increase in γ reduces product differentiation. When $\gamma = 0$, product differentiation is sufficiently large and each firm acts as a monopolist in the market. When $\gamma = 1$, the two goods are perfect substitutes (homogeneous). We exclude these two special cases in this note.

Utility maximization subject to the budget constraint $p_1q_1 + p_2q_2 + z = I$ yields the following inverse demand function of firm i's good in the third country:

$$p_i = P_i(\mathbf{q}) = a - q_i - \gamma q_j \qquad (i, j = 1, 2;\ j \neq i), \tag{1}$$

where p_i is the final product price of firm i's good and $\mathbf{q} = (q_1, q_2)$ denotes the output profile.

For simplicity, we assume that both firms produce their outputs at a constant marginal cost, c, and $c < a$ holds. Let s_i denote the unit export (=export) subsidy provided by country i's government. Using the inverse demand functions in (1), the profit function of firm i is

$$\pi_i(\mathbf{q}, s_i) = (P_i(\mathbf{q}) - c + s_i)\, q_i \qquad (i = 1, 2).$$

Each firm has one owner and one manager and each owner designs an incentive contract to compensate their manager. Following the relative performance scheme in the MP model, we express the incentive contract as the weighed sum of its own profit and the profit relative to that of the rival firm, as below.[1]

$$\begin{aligned} M_i(\mathbf{q}, \mathbf{s}, \theta_i) &= (1 - \theta_i)\pi_i(\mathbf{q}, s_i) + \theta_i[\pi_i(\mathbf{q}, s_i) - \pi_j(\mathbf{q}, s_j)] \\ &= \pi_i(\mathbf{q}, s_i) - \theta_i\, \pi_j(\mathbf{q}, s_j) \qquad (i, j = 1, 2;\ i \neq j), \end{aligned} \tag{2}$$

[1]Miller and Pazgal[6] expressed the incentive contract as the weighed sum of the own firm's profit and the positive rival's profit. We obtain the same results if we replace a positive θ_i with a negative one.

where θ_i denotes the contract term of firm i and $\mathbf{s} = (s_i, s_j)$ denotes the subsidy profile. Because θ_i is the weight owner i places on the negative profit of the rival firm, its value represents the degree of importance of firm's relative performance. The positive and negative values of θ_i indicate the firms' spiteful or altruistic behavior, respectively.

As Matsumura et al.[5] showed, we can interpret the contract term $\theta_i \in (-1, 1)$ as the degree of competitiveness under symmetric Cournot duopolistic competition. The benchmark when $\theta_i = 0$ is the standard Cournot duopoly case. An increasing θ_i strengthens competition and makes firms act as in perfect competition when $\theta_i = 1$. However, reducing θ_i alleviates the competition and induces firms to collude and act as a monopoly when $\theta_i = -1$.

In this paper, we endogenize the owners' delegation decisions and explore a four-stage game. In the first stage, each firm owner decides whether to delegate to a manager. In the second stage, each country's government determines the country-specific production(=export) subsidy rate (s_1, s_2). In the third stage, if the owner decides to delegate, then the owner designs its optimal managerial contract to maximize its profit. In the last stage, the manager (if delegated) or the owner decides the product quantity (q_1, q_2) by competing *à* la Cournot with product differentiation in the third market.

In view of first-stage decisions about $\sigma_i (i = 1, 2) \in \{D, N\}$, where D represents the decision to delegate to a manager and N the decision not to delegate, we can divide our game into four subgames, as Table 1 illustrates. A subgame associated with managerial delegation $(\sigma_1, \sigma_2) (\in \{D, N\} \times \{D, N\})$ is called subgame $\sigma_1\sigma_2$. The payoff $\pi_i^{\sigma_1\sigma_2} (i = 1, 2)$ in the table denotes the equilibrium profit of firm i for subgame $\sigma_1\sigma_2$. In this terminology, subgame NN is *à* la Brander and Spencer [1], in which both owners choose not to delegate a manager, while subgame DD is *à* la Miller and Pazgal(2005) when both owners choose to delegate. We solve the game by backward induction and first consider the subgame DD as the benchmark.

Table 1: Payoff Matrix in the Delegation Game

Firm Profit	Delegation ($\sigma_2 = D$)	Non-Delegation ($\sigma_2 = N$)
Delegation ($\sigma_1 = D$)	π_1^{DD}, π_2^{DD}	π_1^{DN}, π_2^{DN}
Non-Delegation ($\sigma_1 = N$)	π_1^{ND}, π_2^{ND}	π_1^{NN}, π_2^{NN}

3 Firms' Optimal Outputs

In subgame DD, both owners commit to delegate a manager. After observing each government's subsidy rate and each firm's incentive contract, the managers decide their optimal outputs satisfying the following first-order condition (FOC).

$$\begin{aligned} 0 = \frac{\partial M_i(.)}{\partial q_i} &= \frac{\partial \pi_i(.)}{\partial q_i} - \theta_i \frac{\partial \pi_j(.)}{\partial q_i} \\ &= A + s_i - 2q_i - (1 - \theta_i)\gamma q_j, \end{aligned} \tag{3}$$

where $A \overset{\text{def}}{=} a - c > 0$ and $\dfrac{\partial^2 M_i(.)}{\partial q_i^2} < 0$ holds.

We denote $R^i(q_j, \theta_i, s_i)$ as manager i's reaction function as below.

$$R^i(q_j, \theta_i, s_i) = \frac{A + s_i - (1-\theta_i)\gamma q_j}{2}, \tag{4}$$

which yields $R^i_q(.) = \dfrac{\partial R^i(.)}{\partial q_j} = -\dfrac{(1-\theta_i)\gamma}{2}$, $R^i_\theta(.) = \dfrac{\partial R^i(.)}{\partial \theta_i} = \dfrac{\gamma q_j}{2}$, $R^i_s(.) = \dfrac{\partial R^i(.)}{\partial s_i} = \dfrac{1}{2}$.

In view of the value of $R^i_q(.)$, if the two goods are strategic substitutes, then $R^i_q \in (-1, 0)$, such that $1 - 2/\gamma < \theta_i < 1$ is satisfied. Otherwise, if the two goods are strategic complements, then $R^i_q \in (0, 1)$, yielding $1 < \theta_i < 1 + 2/\gamma$. We impose the following assumption.

Assumption 1. *Each firm's optimal output is a strategic substitute to the other's; that is,* $1 - 2/\gamma < \theta_i < 1$ *holds under quantity competition.*

Assumption 1 assures that the equilibrium is globally stable in the standard Cournot output adjustment process below.

$$1 - R^1_q R^2_q = \frac{D}{4} > 0,$$

in which $D \stackrel{\text{def}}{=} 4 - \gamma^2(1-\theta_1)(1-\theta_2) > 0$ is satisfied.

We denote the equilibrium output as $q^E_i(\boldsymbol{s}, \boldsymbol{\theta}) = R^i_q(q^E_j(\boldsymbol{s}, \boldsymbol{\theta}), \theta_i, s_i)$ where $\boldsymbol{\theta} \stackrel{\text{def}}{=} (\theta_1, \theta_2)$ represents the contract term profile. The superscript E represents the output-stage equilibrium values. Given $q_j (j = 1, 2) > 0$, differentiating $q^E_i(\boldsymbol{s}, \boldsymbol{\theta})$ with respect to θ_i yields the following comparative statics results.

$$\frac{\partial q^E_i(\boldsymbol{s}, \boldsymbol{\theta})}{\partial \theta_i} = \frac{4R^i_\theta(.)}{D} = \frac{2\gamma}{D} q_j > 0 \quad , \quad \frac{\partial q^E_j(\boldsymbol{s}, \boldsymbol{\theta})}{\partial \theta_i} = R^j_q(.) \frac{\partial q^E_i(\boldsymbol{s}, \boldsymbol{\theta})}{\partial \theta_i} = -\frac{(1-\theta_j)\gamma^2}{D} q_j < 0 \tag{5}$$

Similarly, differentiating with respect to s_i yields

$$\frac{\partial q^E_i(\boldsymbol{s}, \boldsymbol{\theta})}{\partial s_i} = \frac{4R^i_s(.)}{D} = \frac{2}{D} > 0 \quad , \quad \frac{\partial q^E_j(\boldsymbol{s}, \boldsymbol{\theta})}{\partial s_i} = R^j_q(.) \frac{\partial q^E_i(\boldsymbol{s}, \boldsymbol{\theta})}{\partial s_i} = -\frac{(1-\theta_j)\gamma}{D} < 0.$$

Note that the contract term θ_i has the same effect as the strategic subsidy policy in the BS model. When the owner designs a positive θ_i in the incentive contract, then the manager behaves more aggressively, so the own firm's output increases and the rival's output decreases.

Solving for the FOCs in (3) yields the optimal output of firm i:

$$q^E_i(\boldsymbol{s}, \boldsymbol{\theta}) = \frac{2(A + s_i) - \gamma(1-\theta_i)(A + s_j)}{D}. \tag{6}$$

Setting $\theta_i (i = 1, 2) = 0$ or 1 in (6), we can obtain the corresponding equilibrium output results for the four subgames in Table 1.

4 Government Subsidy Decision

After observing each owner's delegation choice, each government decides the optimal subsidy rate to maximize its own welfare. We assume no domestic consumption in the two exporting countries, so the social welfare function of country $i(i = 1,2)$ is the domestic firm's profit net of the subsidy payment.

$$W_i^E(\boldsymbol{s},\boldsymbol{\theta}) = \pi_i(q_i^E(\boldsymbol{s},\boldsymbol{\theta}), q_j^E(\boldsymbol{s},\boldsymbol{\theta}), s_i) - s_i q_i^E(\boldsymbol{s},\boldsymbol{\theta}) \qquad (i,j = 1,2; j \neq i)$$

4.1 Subgame NN: No delegation, $\boldsymbol{\theta} = (0,0)$

In subgame NN, both owners choose not to delegate a manager. The owners decide the optimal output under pure profit maximization. Setting $\theta_1 = \theta_2 = 0$ in (6), the equilibrium output yields

$$q_i^E(\mathbf{s},\mathbf{0}) = \frac{(2-\gamma)A + 2s_i - \gamma s_j}{4-\gamma^2}.$$

This is *à* la Brander-Spencer [1] with differentiated goods. The rent-shifting subsidy increases domestic production and reduces foreign production.

The FOC of welfare maximization for country i yields

$$0 = \frac{\partial W_i^E(\mathbf{s},\mathbf{0})}{\partial s_i} = -\gamma q_i \frac{\partial q_j^E(\mathbf{s},\mathbf{0})}{\partial s_i} - s_i \frac{\partial q_i^E(\mathbf{s},\mathbf{0})}{\partial s_i}.$$

The first term on the right-hand side represents the domestic profit gain through the rent-shifting effect and the second term represents the subsidy payment loss. Solving for the above FOCs for both countries, we obtain the equilibrium subsidy rate below.

$$s_i^{NN} = \frac{\gamma^2 A}{4+2\gamma-\gamma^2} \qquad (i = 1,2).$$

As both firms produce with the same cost function, we obtain the symmetric market outcomes in subgame NN below.

$$q_i^{NN} = \frac{A+s_i}{2+\gamma} = \frac{2A}{4+2\gamma-\gamma^2} \quad , \quad \pi_i^{NN} = \left(q_i^{NN}\right)^2 = \frac{4A^2}{(4+2\gamma-\gamma^2)^2}$$
$$W_i^{NN} = \pi_i - s_i q_i = \frac{2(2-\gamma^2)A^2}{(4+2\gamma-\gamma^2)^2} \qquad (i = 1,2).$$

4.2 Subgame DD: Both firms delegate, $\boldsymbol{\theta} = (\theta_1, \theta_2)$

In subgame DD, the equilibrium outputs in the output stage of the game are shown by (6).

$$q_i^E(\boldsymbol{s},\boldsymbol{\theta}) = \frac{2(A+s_i) - \gamma(1-\theta_i)(A+s_j)}{D} \qquad (i,j = 1,2;\ j \neq i)$$

Observing both managers' output decisions, the owners decide the optimal contract term of θ_i to maximize their own profit. The maximization problem of firm i reduces to

$$\max_{\theta_i} \ \pi_i^E(\boldsymbol{s},\boldsymbol{\theta}) = \pi_i(q_i^E(\boldsymbol{s},\boldsymbol{\theta}), q_j^E(\boldsymbol{s},\boldsymbol{\theta}), s_i) \qquad (i, j = 1, 2;\ j \neq i). \tag{7}$$

By applying the envelop theorem, we obtain the FOC for firm i's profit maximization in (7).

$$0 = \frac{\partial \pi_i^E(\boldsymbol{s},\boldsymbol{\theta})}{\partial \theta_i} = \frac{\partial \pi_i(.)}{\partial q_i}\frac{\partial q_i^E}{\partial \theta_i} + \frac{\partial \pi_i(.)}{\partial q_j}\frac{\partial q_j^E}{\partial \theta_i} = -\theta_i \gamma q_j \frac{\partial q_i^E}{\partial \theta_i} - \gamma q_i \frac{\partial q_j^E}{\partial \theta_i}. \tag{8}$$

The optimal incentive contract term θ_i in (8) yields [2]

$$\theta_i = -\frac{\gamma q_i \frac{\partial q_j^E}{\partial \theta_i}}{\gamma q_j \frac{\partial q_i^E}{\partial \theta_i}} = -R_q^j \frac{q_i}{q_j} > 0, \tag{9}$$

which is positive according to Assumption 1 and (5). The owners of both firms put positive weight on the relative profit and delegate aggressive managers. The intuition behind is as follows. The positive value of θ_i means that firm owner evaluates not only the absolute profit, but also how the domestic firm can achieve a higher profit than the rival firm. Setting $\theta_i > 0$ induces its manager to behave more aggressively to expand the own firm's output and lower the market price of the domestic good. Meanwhile, the rival firm's output decreases as a strategic substitute good and the rent shifts to the domestic firm. Differentiating $\pi_i^E(.)$ with respect to θ_j in (7) yields

$$\frac{\partial \pi_i^E(\boldsymbol{s},\boldsymbol{\theta})}{\partial \theta_j} = \frac{\partial \pi_i(.)}{\partial q_i}\frac{\partial q_i^E}{\partial \theta_j} + \frac{\partial \pi_i(.)}{\partial q_j}\frac{\partial q_j^E}{\partial \theta_j} = -\theta_i \gamma q_j \frac{\partial q_i^E}{\partial \theta_j} - \gamma q_i \frac{\partial q_j^E}{\partial \theta_j}. \tag{10}$$

Substituting (9) into (10) and using (5), we obtain the equivalent rent-shifting effect of managerial delegation; that is, increasing the incentive contract term reduces the rival firm's profit.

$$\frac{\partial \pi_i^E(\boldsymbol{s},\boldsymbol{\theta})}{\partial \theta_j} = -(1 - R_q^i R_q^j)\gamma q_i \frac{\partial q_j^E}{\partial \theta_j} < 0$$

Solving for the FOCs in (8) within the linear demand function, we obtain each firm's optimal contract term:

$$\theta_i^*(\mathbf{s}) = \frac{\gamma[(A + s_i) - \gamma(A + s_j)]}{2(A + s_j) - \gamma(A + s_i) - \gamma^2(A + s_j)} \qquad (i, j = 1, 2;\ j \neq i),$$

where the superscript $*$ represents the subsidy-stage equilibrium values.

Differentiating $\theta_i^*(\mathbf{s})$ in terms of s_i and s_j yields

$$\frac{\partial \theta_i^*(\mathbf{s})}{\partial s_i} = \frac{2\gamma(1 - \gamma^2)(A + s_j)}{[2(A + s_j) - \gamma(A + s_i) - \gamma^2(A + s_j)]^2} > 0,$$
$$\frac{\partial \theta_i^*(\mathbf{s})}{\partial s_j} = \frac{-2\gamma(1 - \gamma^2)(A + s_i)}{[2(A + s_j) - \gamma(A + s_i) - \gamma^2(A + s_j)]^2} < 0.$$

[2]Note that under symmetric cost functions, $q_i = q_j$, we can simplify $\theta_i = -R_q^j$ and obtain $\theta_i = \theta_j = \frac{\gamma}{\gamma+2} < 1$ in the equilibrium.

An increase in the home government subsidy makes the domestic firm's owner delegate to a more aggressive manager and the rival firm's owner delegate to a more conservative manager. The intuition behind can be explained by de Meza[3], who indicated that under Cournot competition, the low-cost country has the most incentive to encourage its domestic firm to increase its output and leads to higher subsidization. In our model, the contract term θ_i has the equivalent rent-shifting effect as strategic trade policy as explained. The home government subsidy makes the domestic firm more cost efficient and thus encourages the firm owner to expand outputs.

Lemma 1. *An increase in the home government subsidy rate encourages the domestic firm's owner to delegate to a more aggressive manager, while discouraging the same action from the rival firm's owner.*

The resulting equilibrium output is

$$q_i^*(\mathbf{s}) = q_i^E\left(\mathbf{s}, \theta_i^*(\mathbf{s}), \theta_j^*(\mathbf{s})\right) \qquad (i, j = 1, 2;\ j \neq i).$$

Differentiating $q_i^*(\mathbf{s})$ in term of s_i and s_j yields the following results.

$$\begin{aligned}
\frac{\partial q_i^*(\mathbf{s})}{\partial s_i} &= \frac{\partial q_i^E(.)}{\partial s_i} + \frac{\partial q_i^E(.)}{\partial \theta_i}\frac{\partial \theta_i^*(\mathbf{s})}{\partial s_i} + \frac{\partial q_i^E(.)}{\partial \theta_j}\frac{\partial \theta_j^*(\mathbf{s})}{\partial s_i} > 0 \\
\frac{\partial q_j^*(\mathbf{s})}{\partial s_i} &= \frac{\partial q_j^E(.)}{\partial s_i} + \frac{\partial q_j^E(.)}{\partial \theta_i}\frac{\partial \theta_i^*(\mathbf{s})}{\partial s_i} + \frac{\partial q_j^E(.)}{\partial \theta_j}\frac{\partial \theta_j^*(\mathbf{s})}{\partial s_i} < 0 \\
&= R_q^j \frac{\partial q_i^*(\mathbf{s})}{\partial s_i} + (1 - R_q^i R_q^j)\frac{\partial q_j^E(.)}{\partial \theta_j}\frac{\partial \theta_j^*(\mathbf{s})}{\partial s_i} \qquad (11)
\end{aligned}$$

An increase in the home government subsidy expands the domestic equilibrium output through three positive effects: a direct effect by reducing the domestic marginal cost and two indirect effects through the domestic and rival owner's incentive contract determinations. Likewise, the three negative effects affect the foreign firm's equilibrium output.

We can rewrite the welfare function of country i evaluated at the second-stage equilibrium as

$$W_i^*(s) = \pi_i(q_i^*(\mathbf{s}), q_j^*(\mathbf{s}), s_i) - s_i q_i^*(\mathbf{s}).$$

By differentiating $W_i^*(s)$ in terms of s_i, we have

$$\begin{aligned}
\frac{\partial W_i^*(\mathbf{s})}{\partial s_i} &= \frac{\partial \pi_i(.)}{\partial q_i}\frac{\partial q_i^*(\mathbf{s})}{\partial s_i} + \frac{\partial \pi_i(.)}{\partial q_j}\frac{\partial q_j^*(\mathbf{s})}{\partial s_i} - s_i\frac{\partial q_i^*(\mathbf{s})}{\partial s_i}, \qquad (12) \\
&= -\theta_i \gamma q_j \frac{\partial q_i^*(\mathbf{s})}{\partial s_i} - \gamma q_i \frac{\partial q_j^*(\mathbf{s})}{\partial s_i} - s_i\frac{\partial q_i^*(\mathbf{s})}{\partial s_i}, \\
&= -\gamma q_i (1 - R_q^i R_q^j)\frac{\partial q_j^E(.)}{\partial \theta_j}\frac{\partial \theta_j^*(\mathbf{s})}{\partial s_i} - s_i\frac{\partial q_i^*(\mathbf{s})}{\partial s_i}. \qquad (13)
\end{aligned}$$

The first and second terms on the right-hand side of (12) represent the domestic profit loss and gain through the combination of managerial delegation and government subsidization, respectively. The rent-shifting effects of strategic managerial delegation and government subsidization expands domestic output and reduces the rival's output. The domestic firm benefits from gaining a larger market share. However, excessive production lowers the market price and reduces the firm's profit. By substituting (11), we can rewrite the FOC as (13) and find that the positive rent-shifting effect outweighs the negative excessive production effect. The third term represents the subsidy payment loss. Setting $s_i = 0$, we have $\frac{\partial W_i^*(\mathbf{s})}{\partial s_i}|_{s_i=0} > 0$, implying that each government has an incentive to subsidize its own outputs.

Solving for the FOCs for both countries, we obtain the equilibrium subsidy rate:

$$s_i^{DD} = \frac{\gamma^3 A}{8 - 4\gamma^2 - \gamma^3} > 0.$$

Under a symmetric cost structure, we obtain the following equilibrium market outcome in the subgame DD.

$$\begin{aligned}
\theta_i^{DD} &= \frac{\gamma(\gamma-1)(A+s_i)}{(\gamma+2)(\gamma-1)(A+s_i)} = \frac{\gamma}{\gamma+2} \in \left(\frac{1}{3}, 1\right) \\
q_i^{DD} &= \frac{[2-\gamma(1+\theta_i)](A+s_i)}{4-\gamma^2(1+\theta_i)^2} = \frac{(2-\gamma^2)A}{(\gamma+1)(4-2\gamma-\gamma^2)} \\
\pi_i^{DD} &= [A-(1+\gamma)q_i+s_i]q_i = \frac{(2-\gamma)(2-\gamma^2)^2A^2}{(\gamma+1)(\gamma+2)(4-2\gamma-\gamma^2)^2} \\
W_i^{DD} &= [A-(1+\gamma)q_i]q_i = \frac{2(1-\gamma)(2-\gamma^2)A^2}{(1+\gamma)(4-2\gamma-\gamma^2)^2}
\end{aligned}$$

4.3 Subgame DN & ND: Unilateral delegation, $\boldsymbol{\theta} = (\theta_1, 0), (0, \theta_2)$

Next, we consider subgame DN, which is the unilateral delegation case in which only firm 1 delegates a manager and firm 2 does not. At the output stage, firm 1's owner decides the output under the pre-committed contract, while firm 2's owner decides the output as a pure profit maximizer. The FOCs are $\frac{\partial M_1(\mathbf{q},\mathbf{s},\theta_1)}{\partial q_1} = 0$ and $\frac{\partial \pi_2(\mathbf{q},s_2)}{\partial q_2} = 0$. Setting $\theta_2 = 0$ in (6), we derive the output-stage equilibrium outputs:

$$\begin{aligned}
q_1^E(\boldsymbol{s}, \theta_1) &= \frac{2(A+s_1) - \gamma(1-\theta_1)(A+s_2)}{D'} \\
q_2^E(\boldsymbol{s}, \theta_1) &= \frac{2(A+s_2) - \gamma(A+s_1)}{D'},
\end{aligned}$$

where $D' = 4 - \gamma^2(1+\theta_1)$.

In the delegation stage, only firm 1's owner designs an incentive contract to a manager. We can rewrite firm 1's profit function as $\pi_1^E(\mathbf{s}, \theta_1) = \pi_1(q_1^E(\boldsymbol{s}, \theta_1), q_2^E(\boldsymbol{s}, \theta_1), s_1)$. Differentiating with respect to θ_1 and solving for the FOC in (8), we obtain the optimal contract term of firm 1:

$$\theta_1^*(\mathbf{s}) = -R_q^2 \frac{q_1}{q_2} = -\frac{\gamma[2(A+s_1) - \gamma(A+s_2)]}{(4-\gamma^2)(A+s_2) - 2\gamma(A+s_1)}.$$

As for the equilibrium output, we can rewrite $q_i^*(\mathbf{s}) = q_i^E\left(\mathbf{s}, \theta_1^*(\mathbf{s})\right)$. Differentiating $q_i^*(\mathbf{s})$ with respect to s_i yields the output change below. Comparing to the output change results in the subgame DD, the same rent-shifting effect of subsidization holds. Since only firm 1 chooses delegation, the indirect effect through firm 2's managerial delegation does not exist.

$$\begin{aligned} \frac{\partial q_i^*(\mathbf{s})}{\partial s_i} &= \frac{\partial q_i^E}{\partial s_i} + \frac{\partial q_i^E}{\partial \theta_1}\frac{\partial \theta_1^*}{\partial s_i} > 0 \\ \frac{\partial q_i^*(\mathbf{s})}{\partial s_j} &= \frac{\partial q_i^E}{\partial s_j} + \frac{\partial q_i^E}{\partial \theta_1}\frac{\partial \theta_1^*}{\partial s_j} = R_q^i \frac{\partial q_j^*(\mathbf{s})}{\partial s_j} < 0 \end{aligned} \tag{14}$$

We can derive the FOC for country 1's welfare maximization

$$\begin{aligned} \left.\frac{\partial W_1^*(\mathbf{s})}{\partial s_1}\right|_{s_1=0} &= \frac{\partial \pi_1(.)}{\partial q_1}\frac{\partial q_1^*(\mathbf{s})}{\partial s_1} + \frac{\partial \pi_1(.)}{\partial q_2}\frac{\partial q_2^*(\mathbf{s})}{\partial s_1} + s_1\frac{\partial q_1^*(\mathbf{s})}{\partial s_1} \\ &= \left(\theta_1 q_2 - q_1 R_q^2\right)\gamma\frac{\partial q_1^*(\mathbf{s})}{\partial s_1} = 0 \end{aligned}$$

by using (14). As firm 1's equilibrium contract result yields $\theta_1 = R_q^2 q_1/q_2$, we find $\left.\frac{\partial W_1^*(\mathbf{s})}{\partial s_1}\right|_{s_1=0} = 0$. Thus, Country 1's government has no incentive to subsidize its own output, i.e.,

$$s_1^{DN} = 0.$$

We have the same intuition as in Wei[8]. When only firm 1 delegates a manager, the aggressive incentive contract expand firm 1's output. Home government subsidization further leads to excessive production and reduces firm 1's profit, which is a part of social welfare. Hence, country 1's government has no incentive to intervene.

By applying the envelop theorem, the FOC for country 2's welfare maximization yields

$$\begin{aligned} \left.\frac{\partial W_2^*(\mathbf{s})}{\partial s_2}\right|_{s_2=0} &= \frac{\partial \pi_2(.)}{\partial q_1}\frac{\partial q_1^*(\mathbf{s})}{\partial s_2} + s_2\frac{\partial q_2^*(\mathbf{s})}{\partial s_2} \\ &= -\gamma q_2\frac{\partial q_1^*(\mathbf{s})}{\partial s_2} > 0. \end{aligned}$$

Country 2's government has a positive incentive to subsidize its outputs. We obtain the equilibrium subsidy rate of country 2 as

$$s_2^{DN} = \frac{\gamma^2(4-2\gamma-\gamma^2)A}{(4-\gamma^2)(4-3\gamma^2)} > 0.$$

The corresponding symmetric equilibria yield the following results.

$$\begin{aligned} \theta_1^{DN} &= \frac{\gamma[2A-\gamma(A+s_2)]}{(4-\gamma^2)(A+s_2)-2\gamma A} = \frac{\gamma(8-4\gamma-4\gamma^2+\gamma^3)}{(2+\gamma)(2-\gamma)(4-2\gamma-\gamma^2)} > 0 \\ q_1^{DN} &= \frac{2A-\gamma(1+\theta_1)(A+s_2)}{4-\gamma^2(1+\theta_1)} = \frac{(8-4\gamma-4\gamma^2+\gamma^3)A}{(4-\gamma^2)(4-3\gamma^2)} \\ q_2^{DN} &= \frac{2(A+s_2)-\gamma A}{4-\gamma^2(1+\theta_1)} = \frac{(4-2\gamma-\gamma^2)A}{2(4-3\gamma^2)} \end{aligned}$$

$$\pi_1^{DN} = (A - q_1 - \gamma q_2)q_1 = \frac{(2-\gamma^2)(8-4\gamma-4\gamma^2+\gamma^3)^2A^2}{2(4-\gamma^2)^2(4-3\gamma^2)^2} = W_1^{DN}$$

$$\pi_2^{DN} = \left(q_2^{DN}\right)^2 = \frac{(4-2\gamma-\gamma^2)^2A^2}{4(4-3\gamma^2)^2}, \qquad W_2^{DN} = \pi_2 - s_2q_2 = \frac{(4-2\gamma-\gamma^2)^2A^2}{4(4-\gamma^2)(4-3\gamma^2)}$$

Meanwhile, under a symmetric cost function, we can solve subgame ND as follows.

$$\Omega_i^{DN} = \Omega_j^{ND}, \qquad i, j = 1, 2; j \neq i,$$

where $\Omega = \{p, q, s, \pi, W\}$ represents the corresponding equilibrium values.

5 Managerial Delegation Game

In the previous section, we analyzed the market equilibria under each delegation structure. Before examining the owners' delegation choices in the first stage, we first compare the equilibrium subsidy rates under the four subgames.

Lemma 2. *(i) Irrespective of the value of γ, the country with unilateral delegation always results in the free trade equilibrium, while the country with unilateral non-delegation results in the highest subsidy rate.*

(ii) If $\gamma > \hat{\gamma} \equiv \frac{\sqrt{13}-1}{3}$, $s_1^{ND} > s_1^{DD} > s_1^{NN} > s_1^{DN} = 0$ holds. Otherwise, if $\gamma < \hat{\gamma}$, then $s_1^{ND} > s_1^{NN} > s_1^{DD} > s_1^{DN} = 0$ holds.

Proof. See Appendix. □

Wei[8] offers an explanation for Lemma 2(i). When only the domestic firm commits to delegate a manager, the domestic government lacks an incentive to subsidize, while the rival country's government has a stronger subsidization incentive, acting as a Stackelberg leader in the subsidy competition. Lemma 2(ii) shows that if the two goods are very close substitutes with larger γ, the equilibrium subsidy rate is higher when both firms choose delegation than when neither firm choose delegation. Das [2] showed that the presence of managerial delegation weakens the country's subsidization incentive under the FJS contract model with homogenous goods. Under the MP contract model with differentiated goods in this paper, the effect of managerial delegation on the government's subsidization incentive depends on the degree of product differentiation.

Lemma 3. *Irrespective the value of γ, $q_1^{DD} > q_1^{ND} > q_1^{NN} > q_1^{DN}$ always holds.*

Proof. See Appendix. □

Lemma 3 shows the ranking of the equilibrium outputs in the four subgames. As both managerial delegation and subsidization expands the own firm's output, subgame DD yields the largest equilibrium output. The ranking of equilibrium outputs in the other three subgames follows the ranking of optimal subsidy results in Lemma 2.

We now turn to the firms' endogenous determination of delegation in the first stage. We summarize the firms' profits in Table 1. We can show the firms' equilibrium profits in terms of γ under the four subgames as in Figure 1 in the Appendix by using MATLAB. We obtain the following results.

$$\pi_1^{DD} < \pi_1^{ND} \quad , \quad \pi_1^{DN} < \pi_1^{NN}$$

This result implies that non-delegation is a dominant strategy for both firms under a symmetric cost structure. Comparing π_i^{DD} with π_i^{NN}, we find that the Pareto-efficient equilibrium does not always occur. When γ becomes very close to 1, the game ends in a prisoner's dilemma.

Proposition 1. *When firm owners commit to delegate a manager or not prior to the governments' subsidy decision, both owners choose not to delegate a manager and the Pareto-efficient equilibrium can occur, except if the two goods are near perfect substitutes (γ is close to 1). Furthermore, the social optimum result is realized.*

Proof. See Appendix. □

The intuition behind Proposition 1 is as follows. In view of Lemma 2, $s_1^{NN} > s_1^{DN}$ and $s_1^{ND} > s_1^{DD}$ hold. Given the rival firm's delegation decision, choosing non-delegation always induces home government to provide a higher subsidy, thus increasing the own firm's profit. Predicting the governments' decisions in the second stage, both owners choose not to delegate in the first stage. When γ is large and close to 1, $s_1^{DD} > s_1^{NN}$ in Lemma 2. When both firms delegate to a manager, they can obtain higher subsidies and achieve higher profit than when both do not delegate. However, the game equilibrium may result in a prisoner's dilemma due to excess competition.

6 Conclusions

In this note, we adopt the relative performance incentive contract following the MP contract model and reexamine the implications of the separation of ownership and management with an export subsidy policy in a third market. We consider horizontal product differentiation and symmetric costs. In this paper, we discuss the nature of the equivalent strategic behavior between government trade policy and managerial delegation under quantity competition. We derive some results similar to Wei[8], who examined the FJS contract model. Endogenizing the firms' managerial delegation decisions, we find that both firms choose not to delegate a manager and the outcome is the socially optimum result. However, the Pareto-efficient equilibrium is not satisfied when the two goods are very close substitutes.

This note does not consider asymmetric cost and import competition. The symmetric assumption and third-market setting simplify the analysis, but offer few implications for the cost difference and domestic consumption. In addition, the equivalence of price and quantity competition in the MP model corresponds to a bilateral delegation structure, but not to unilateral delegation. It is challenging to examine how the mode of market competition affects the equilibrium results under unilateral delegation. We leave these challenges for future research.

Appendix

Proof of Lemma 2

Using the equilibrium subsidy results in Section 4, we obtain

$$s_1^{ND} - s_1^{NN} = \frac{\gamma^2(4-2\gamma-\gamma^2)A}{16-16\gamma^2+3\gamma^4} - \frac{\gamma^2 A}{4+2\gamma-\gamma^2} = \frac{2\gamma^4(2-\gamma^2)}{(16-16\gamma^2+3\gamma^4)(4+2\gamma-\gamma^2)} > 0$$

$$s_1^{ND} - s_1^{DD} = \frac{\gamma^2(4-2\gamma-\gamma^2)A}{16-16\gamma^2+3\gamma^4} - \frac{\gamma^3 A}{8-4\gamma^2-\gamma^3} = \frac{2\gamma^2(1-\gamma)(8-4\gamma-4\gamma^2-\gamma^3)}{(16-16\gamma^2+3\gamma^4)(8-4\gamma^2-\gamma^3)} > 0$$

$$s_1^{DD} - s_1^{NN} = \frac{\gamma^3 A}{8-4\gamma^2-\gamma^3} - \frac{\gamma^2 A}{4+2\gamma-\gamma^2} = \frac{2\gamma^2(3\gamma^2+2\gamma-4)}{(4+2\gamma-\gamma^2)(8-4\gamma^2-\gamma^3)},$$

which yields $s_1^{ND} > s_1^{NN}$ and $s_1^{ND} > s_1^{DD}$. To compare s_1^{DD} and s_1^{NN}, we solve for $3\gamma^2 + 2\gamma - 4 = 0$. We denote $\hat{\gamma} \overset{\text{def}}{=} \frac{\sqrt{13}-1}{3}$. If $\gamma > \hat{\gamma}$, then $s_1^{DD} > s_1^{NN}$ holds, and vice versa if $\gamma < \hat{\gamma}$. Then, we can obtain the ranking of equilibrium subsidy rates in Lemma 2.

Proof of Lemma 3

Using the equilibrium output results in Section 4, we obtain

$$q_1^{DD} - q_1^{ND} = \frac{(2-\gamma^2)A}{(\gamma+1)(4-2\gamma-\gamma^2)} - \frac{(4-2\gamma-\gamma^2)A}{2(4-3\gamma^2)} = \frac{\gamma^4(1-\gamma)A}{2(1+\gamma)(4-3\gamma^2)(4-2\gamma-\gamma^2)} > 0$$

$$q_1^{ND} - q_1^{NN} = \frac{(4-2\gamma-\gamma^2)A}{2(4-3\gamma^2)} - \frac{2A}{4+2\gamma-\gamma^2} = \frac{\gamma^4 A}{2(4-3\gamma^2)(4+2\gamma-\gamma^2)} > 0$$

$$q_1^{NN} - q_1^{DN} = \frac{2A}{4+2\gamma-\gamma^2} - \frac{(8-4\gamma-4\gamma^2+\gamma^3)A}{(4-\gamma^2)(4-3\gamma^2)} = \frac{\gamma^5 A}{(4+2\gamma-\gamma^2)(4-\gamma^2)(4-3\gamma^2)} > 0,$$

which yields $q_1^{DD} > q_1^{ND} > q_1^{NN} > q_1^{DN}$.

Proof of Proposition 1

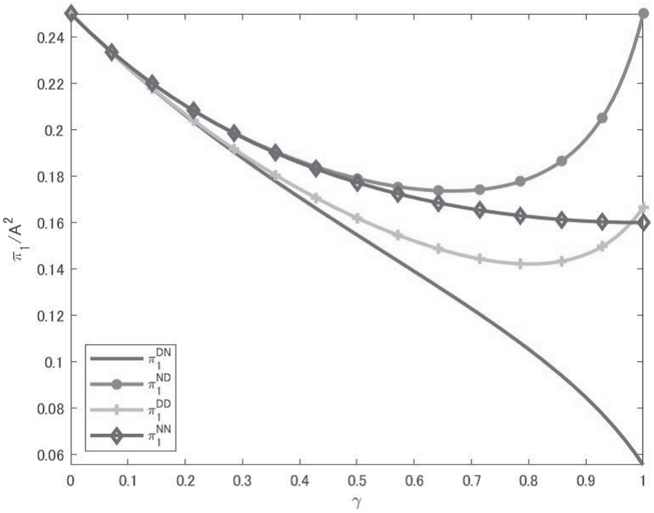
In view of firms' endogenous delegation decision in Table 1, we compare firm 1's equilibrium profit given firm 2's delegation decision.

$$\begin{aligned}
\pi_1^{DD} - \pi_1^{ND} = \pi_1^{DD} - \pi_2^{DN} &= \frac{(2-\gamma)(2-\gamma^2)^2 A^2}{(\gamma+1)(\gamma+2)(4-2\gamma-\gamma^2)^2} - \frac{(4-2\gamma-\gamma^2)^2 A^2}{4(4-3\gamma^2)^2} \\
&= \frac{-\gamma^2 \Psi_1(\gamma)\, A^2}{4(\gamma+1)(\gamma+2)(4-2\gamma-\gamma^2)^2(4-3\gamma^2)^2} < 0 \\
\pi_1^{DN} - \pi_1^{NN} &= \frac{(2-\gamma^2)(8-4\gamma-4\gamma^2+\gamma^3)^2 A^2}{2(4-\gamma^2)^2(4-3\gamma^2)^2} - \frac{4A^2}{(4+2\gamma-\gamma^2)^2} \\
&= \frac{-\gamma^2 \Psi_2(\gamma)\, A^2}{2(4-\gamma^2)^2(4-3\gamma^2)^2(4+2\gamma-\gamma^2)^2} < 0,
\end{aligned}$$

where $\Psi_1(\gamma) = 256 - 256\gamma - 448\gamma^2 + 480\gamma^3 + 224\gamma^4 - 264\gamma^5 - 38\gamma^6 + 47\gamma^7 + \gamma^8 > 0$ and $\Psi_2(\gamma) = 1024 - 2048\gamma^2 + 128\gamma^3 + 1408\gamma^4 - 192\gamma^5 - 384\gamma^6 + 88\gamma^7 + 34\gamma^8 - 12\gamma^9 + \gamma^{10} > 0$.

The figure below illustrates firm 1's profits in terms of $\gamma \in (0, 1)$ in the four subgames by using MATLAB.

Figure 1: Firm 1's Profits



For the social welfare in subgames DD and NN, we obtain

$$\begin{aligned} W_1^{DD} - W_1^{NN} &= \frac{2(1-\gamma)(2-\gamma^2)A^2}{(1+\gamma)(4-2\gamma-\gamma^2)^2} - \frac{2(2-\gamma^2)A^2}{(4+2\gamma-\gamma^2)^2} \\ &= \frac{-4\Psi_2(\gamma)A^2}{(1+\gamma)(4-2\gamma-\gamma^2)^2(4+2\gamma-\gamma^2)^2} < 0, \end{aligned}$$

where $\Psi_3(\gamma) = 16-32\gamma-4\gamma^2+40\gamma^3-15\gamma^4-7\gamma^5+3\gamma^6 > 0$. The above result yields $W_1^{NN} > W_1^{DD}$, implying that the social optimum result is realized under subgame *NN*.

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